AN ELEMENTARY MANUAL OF STATISTICS

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PREFACE TO FOURTH EDITION

THE changes in the nature, contents and presentation of official and other statistics since 1914 have been so extensive, that it has been necessary to re-cast and re-write most of the sections of Part II. The matter relating to pre-war statistics has been preserved wherever it seemed still to have interest, so that comparison is possible in respect of the nature of the sources of information and of the definitions used before and after the War, while the tables are arranged so that consecutive views of the changes in trade, prices, wages, etc., can be obtained over long periods.

The general scope has been widened so as to include summary statistics of population, production, prices, etc., in the United States, especially when useful comparison can be made with those of the United Kingdom.

No alterations have been made in Part I in this edition, because the relevant methods of statistics are unchanged, and if it is remembered that the illustrations are based on the pre-war scale of values a certain interest still attaches to them.

July 1928.

PREFACE TO FIRST EDITION

THIS manual is intended for the use of those who desire some knowledge of statistical methods and statistical results without going deeply into technicalities or undertaking mathematical analysis, it is hoped that it will be of service to all who have occasion to use statistics in their own business or profession, or who take an intelligent interest in public affairs.

It is also designed as a first course in statistics for students who wish to proceed further in the subject and, if it serves its purpose, will stimulate interest in the many fascinating problems that await solution, and that can only be attacked by the methods of modern mathematical statistics.

The first part deals with elementary methods and with such technical terms and ideas as are indispensable in the handling of numbers on a large scale. In the second part the origin of many groups of public statistics is shown, their adequacy is criticized, and some of the more interesting results which are based on them are briefly summarized. This part is intended as a guide to official statistics, not as a compendium or dictionary of them; and the problems attacked are given rather as illustrations than as substantial contributions to knowledge.

To facilitate the use of the book in the hands of teachers a number of exercises of various degrees of complexity are given in Appendix I. Every serious student of commercial or public affairs should be acquainted with the nature of the contents of the Statistical Abstract of the United Kingdom; to promote this knowledge, and because many pages of headlines and figures would otherwise have been necessary, a large proportion of the examples relate to tables in the Abstract for 1909,* for which future or earlier abstracts can readily be substituted.

Appendix II contains a short list of Blue Books which should be easily accessible in the Library of every institution where the subject of statistics has a place in the curriculum.

My thanks are due to Dr. Dudfield (Medical Officer of Health for Paddington) and Professor Cannan for most useful criticism of some of the chapters, and to Mr. G. W. Palmer for help and advice in the correction of proofs. I shall be grateful for any criticisms which will tend to increase the utility or improve the accuracy of the book.

Reading,
December 1909.

A. L. B.

1926 in fourth edition.

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PART I

AN ELEMENTARY MANUAL OF STATISTICS

CHAPTER I

NATURE AND USE OF STATISTICS

1. STATISTICS are numerical statements of facts in any department of inquiry, placed in relation to each other; statistical methods are devices for abbreviating and classifying the statements and making clear the relations. The elementary methods are based on arithmetical processes of an easy but specialized kind; more refined methods, necessary for certain classes of investigation, involve complex mathematical ideas.

2. Statistical treatment is necessary in a very great variety of cases, some of which may be distinguished as follows—

Groups.—If a large number of things or persons have something in common, *e. g.* as members of the same nation, workers in the same occupation, houses in a defined locality, but differ one from the other in respect to some measurable characteristic, *e. g.* age, amount of wages, rateable value, together they form a statistical group. Groups can be represented by *diagrams*, *tabulated* in grades, or described in abbreviated form by *averages*.

Classes.—If the characteristics in which the things or persons differ are not measurable, but need separate description, *e. g.* the number of persons in different districts, or in different occupations in the same industry, or of houses used for

different purposes, a statistical *table* can be made showing in juxtaposition the numbers in various classes and sections according to any scheme of classification, and the relative sizes of the classes can be indicated by *percentages*.

Series.—If the numbers in some group or class are counted, or the quantities or values of some aggregate are measured, periodically (weekly, monthly or annually), we obtain a statistical series, whose nature is most easily appreciated with the help of a diagram.

3. Statistics are thus used for describing and analyzing large groups or aggregates, too large or complex to be intelligible by simple observation. Thus the affairs of a community, the progress of a large business, and the productivity of a country need statistical treatment, while the individual, the single transaction, the quantity grown in a field do not. The difference is not one of degree only, for when investigation is extended over a large area, regularity is obtained, conformity to general laws is visible, and new methods of description are required, while observation of a few cases suggests only chance and chaos. There is infinite variety in the constitution of a family, but in a community the distribution by age is nearly invariable. Men differ from each other in stature and in wealth; but simple mathematical formulæ describe the distribution as to height and as to income of the members of a nation. Statistics generalize and repair the defects of individual experience.

4. Statistics are specially useful for making *comparisons* of similar aggregates from time to time, or from place to place. The significance of one quantity, *e.g.* the average wage of a group of workmen, can only be appreciated by comparison with another, *e.g.* the average wage of another group in a different occupation or district, or the same group at an earlier date. The gradual *change* of the birth-, death- or marriage-rates during a series of years, shows very much more than the statements for a single year. Again, it is frequently necessary to show the *relation* of one quantity—for example, the total importation of wheat—to another, for

example, the population; or, to take another instance, the relation of the total wheat crop to the area under cultivation. The choice and exact definition of the aggregates that should be thus brought in relation to each other are by no means simple matters.

5. When observations are thus extended, many sources of inaccuracy are found to be present, and it is very frequently impossible to remove them completely. Statistical results are, therefore, very generally estimates rather than exact statements, and it is a matter of the very greatest importance to learn to what degree of accuracy various statements can be trusted, and to obtain methods of neutralizing the effects of errors and omissions of all kinds.

6. Perhaps the principal cause of incorrect use of statistics is want of attention to the definition, meaning and limitation of each estimate quoted. A total, such as the population of England and Wales, or the total value of goods imported into the United Kingdom, is generally the result of a complicated system of enumeration, in which a large body of persons have co-operated, working under printed instructions. To know what is included in the total implies not only careful reading of the title, "Total value of Foreign and Colonial Merchandise Imported," but also knowledge of the method of valuation, of the definitions of "Merchandise" and of "Imported," and of the nature of the omissions (goods brought in as personal luggage or smuggled, etc.). There is hardly any total whose full meaning is apparent simply from its description; there is always to be implied some such phrase as "so far as the items are included in the working definition and enumerated by the staff concerned." The total or average used is a total or average of many items, each of which satisfies some complex definition; this definition is not thoroughly known till the whole method of collection and tabulation is known. In many cases the necessary explanations are given in the introduction to or the footnotes of an official report; in others, where information is not forthcoming, extreme caution is necessary in

using the figures, till a careful inquiry as to their meaning can be made. In Part II below, some of the more important definitions are given.

7. It is frequently the case that the quantity as to which knowledge is desired is not capable of numerical measurements. We cannot measure health, poverty or crime; we can only measure the death-rate, and count the number of persons who receive public relief, and the number of convictions. In such cases the measurements can only be used as indications, and their relation to the more important quantity must be constantly criticized, while other indications should be obtained wherever possible to check the impressions formed. Thus the number of paupers changes with altered administration, and of criminals with modifications of law, and the death-rate differs with age, sex, locality and occupation; but in these cases we have other means of knowing and testing the changes in the quantities concerned.

8. It is very important to avoid mistaking the part for the whole. The growth of exports is often used as an indication of the general growth of trade, but the more important home trade has not been measured, and the whole may diminish while exports increase, or *vice versa*. The number of members of certain trade unions out of work is published monthly, but the percentages based on them cannot be used to measure unemployment as a whole without many qualifications. If our definitions are correct they will show the limitations in extent of our estimates. Other cautions as to common mistakes in using statistics will be found scattered through the chapters that follow.

9. Three of the principal uses of statistics are (i) to give correct views, based on facts, as to what has happened in the past; how, when and under what circumstances, population, trade, wealth, etc., have grown; and by comparison and analysis to search for the causes of changes that have taken place; (ii) to afford material for estimates for the present, *e.g.* the probable yield of a new tax, the amount of trade that will be carried by a new route, the quantity of water needed

by a town; (iii) to make possible a forecast for the near future; for this purpose we study the changes that have taken place in the recent past, by the light of the relations between phenomena that comparative statistical analysis reveals.

10. The main sources of statistical information are (i) official tables published periodically by various Government Departments, (ii) the results of special inquiries made by the Departments or by Royal Commissions or Parliamentary Committees, (iii) monthly and annual reports on special trades made by Chambers of Commerce, trade newspapers and private firms; (iv) special investigations made by private individuals as to social conditions. All of these have their limitations and present special difficulties, and together they are quite inadequate to afford sufficient information as to most of the conditions of welfare, progress and trade which form the subjects of inquiry. There is urgent need for more systematic and more complete national statistics.

11. Even if statistics were complete and perfect, their use would be definitely limited to one aspect of a problem, that is, the numerical aspect. Statistical results are essential, when judgment is to be formed on any questions that involve numbers, quantities or values, but they must always be brought into relation with the personal, political, æsthetic or other non-quantitative considerations that may be of greater importance in deciding on a course of action. Statistics only furnish a tool, necessary though imperfect, which is dangerous in the hands of those who do not know its use and deficiencies. A knowledge of methods and limitations is necessary, if only to avoid being misled by unscrupulous or unscientific arguments.

CHAPTER II

ACCURACY AND APPROXIMATION

1. PERFECT accuracy is very seldom obtained in statistics and in this respect they differ from accountancy. A statement of fact involving £ *s.* *d.* can be made exactly, and must be so made to afford a perfect balance, but as soon as we deal with quantities or values, where the things counted are not perfectly similar to each other or are matters of estimate, we can no longer give an exact unqualified statement. There is no means of knowing exactly the quantity of wheat grown in the United Kingdom, both because one bushel of wheat differs from another in dryness, fineness, and other respects, and because the whole bulk is not and cannot be measured, but is estimated from the acreage under wheat and the average productivity. We cannot know the population of England and Wales exactly on June 30, 1909, for it is eight years since the population was counted; no record is kept as to the numbers who have gone to or come from other parts of the United Kingdom, statistics of emigration to and immigration from the colonies and foreign countries are imperfect, and probably a small number of births are unregistered. Both these totals can, however, be estimated with considerable accuracy.

2. In such cases we should not say that the population consists of 35,751,963 persons, or that 56,531,198 bushels of wheat were produced in 1907, except as bare numerical results of a calculation; but we should aim at finding to how many figures the statements are likely to be correct.

Supposing this difficult operation performed, and that (for example) an error of 100,000 persons is possible in the

population and of 250,000 bushels in the estimated production of wheat, various methods of statement are open to us.

(*a*) The population is 35,751,963 $\pm$ a number not greater than 100,000.

(*b*) The population is 35,750,000 $\pm$ 100,000, or 3575 $\pm$ 10 (0000's omitted).

(*c*) The population is between 35,650,000 and 35,850,000.

(*d*) The population is 36×10^6, or 36,000,000, to the nearest million; or (in a table involving other similar figures) is 36 (000,000's omitted).

(*e*) The population is 35,750,000, correct to ·3%, or to 3‰.

(*f*) The population is $35^{65}_{85} \times 10^4$ (where $^{65}_{85}$ is not a fraction, but an abbreviation for "between 65 and 85").

If the error were, however, known to be not more than 2,000, we could make a shorter statement, viz. that the population is 35,750,000 "in round numbers" or "correct to 10,000"; for the maximum and minimum possible, viz. 35,753,963 and 35,749,963, are both nearer to 35,75 than to 35,74 or 35,76 (0000's omitted). This is the best method when applicable, but in the case given we cannot be sure which is the nearest 100,000, and (*d*), which is the corresponding statement, is unnecessarily rough.

Each of the above statements would be correct for some purpose; the choice depends on the nature of the table of which it is to form part. (*c*) is the clearest if we are not making a table. (*e*), or an equivalent form, is the most scientific. (*f*) has not actually come into use, but may be suggested as the most compact way in which the whole data can be stated.

3. When round numbers are used, the last digit retained must be the nearest to the estimate, not the next under.

Thus 374,563 is $374{,}56^0$ or 3746^{00} or 375^{000} * or 37^{0000}, not 3745^{00} and 374^{000}. In the third case, the number being

* This is merely a convenient way of writing 375,000, when it is implied that the number is correctly given only as far as the 5.

nearly midway between 374,000 and 375,000, it would be better to write 374·5 (thousands). Round numbers are employed both as abbreviations of nearly exact statements and to indicate the accuracy of estimates, the last digit given being supposed correct.

*4. The arithmetic of inexact numbers needs special attention. Unnecessary work is to be avoided; only those digits should be given in the result which are supported by the premises; an indication of the possible error should be given. The following five examples show various ways in which the work can be carried out.**

Addition.—Add 47,386, 9,453, 843,782, the numbers being correct to 2 %, 5 %, and ·5% respectively.

Then the first number is only given as between 47,386 + 948 and 47,386 − 948, and the work may be set down as follows:—

$$\begin{array}{r} 47{,}386 \pm 948 \\ 9{,}453 \pm 473 \\ 843{,}782 \pm 4{,}219 \\ \hline 900{,}621 \pm 5{,}640 \end{array}$$

Answer, 900,000, correct to ·6 %, or "between 895,000 and 906,000."

If a less exact answer is sufficient, we may notice that the last entry makes the greatest contribution to the error, and write

$$\begin{array}{rl} 47 & \text{000's omitted.} \\ 9 & \\ 84 & \\ \hline 90 \times 10^4 & \end{array}$$

Subtraction.—Subtract £85,460 from £197,000, the numbers being correct to the last digit (other than 0) given.

Then the first quantity is only given as between £85,455 and £85,465, the second as between 197,500 and 196,500.

* The concise statement given in these paragraphs will, it is hoped, be sufficient for capable arithmeticians, and a fuller treatment would be out of place; but these or similar methods are to be found in modern Arithmetics, to which the reader is referred if the ideas are not clear. In the end every one makes his own rules for abbreviation.

Work showing

the Maximum difference		the Minimum difference	
197,500			196,500
85,455			85,465
£112,045			£111,035

Answer, £11$\frac{1}{2}$ × 10^3, or £111,500, correct to ·5 %.

Multiplication.—Multiply £30 18*s.* 6*d.* by 347,100, the numbers being correct to the nearest 6*d.* and the nearest 100 respectively.

Greatest possible errors:—3*d.* in £31, or 1 in 2480; and 50 in 347,100, or 1 in 7000.

First method.

Product if there were no error.	Maximum product.
£30·9$\frac{1}{2}$$\frac{2}{4}$5	£30·9$\frac{1}{2}$$\frac{3}{4}$7
347,100	347,150
92775	92811
12370	12375
2165	2166
31	31
	15
£10,734,1^{00}	£10,739,8^{00}

Maximum error ± 5,700 or ·53, %$_{\circ}$, where ·53 %$_{\circ}$ stands for ·53 per mille.

Second method.—Observe that the answer can only be correct to four significant figures.

The maximum errors in the factors are ·40 %$_{\circ}$ and ·13 %$_{\circ}$. Where small percentage errors occur in factors, both being in excess or both in defect, it is easily shown by algebra or geometry that the error in the product is the sum of the errors in the factors. The product is therefore subject to an error of ·53 %$_{\circ}$.

3$\frac{1}{2}$$\frac{3}{4}$1

3·0925 × 10^3

10,413
312
7
1

£10,733 × 10^3

Answer, £10,733,^{000}, correct to ·5 %$_{\circ}$ or £107$\frac{1}{2}$ × 10^4.

Third method.—A little experience will show that the

more serious error comes from the first term and is roughly ·4 ‰. The work should then be done to five figures, and the answer given as doubtful to one unit in the fourth figure.

Division.—45,340,000 tons are valued at £74,380,000. Find the value per ton, the numbers being correct to the last digit (not 0) stated.

First method.—The maximum error is obtained when the dividend is greatest and the divisor least, or *vice versa*.

Maximum possible value.	Value if there were no error.
45,335)74,385(£1·6408	45,340)74,380(£1·6405
45,335	45,340
29,050	29,040
27,201	27,204
1,849	1,836
1,813	1,814
36	22

Answer, £1 12*s.* 9¾*d.*, to nearest farthing.

Second method.—The maximum errors are 1 in 9,000 and 1 in 15,000; if cumulative they make ·11 + ·07 = ·18 ‰, that is ⅛ of one farthing in £1. The quotient, worked on the supposition that there is no error, is therefore correct to the nearest farthing.

Square root.—Find the length of the side of a square field whose area is 15 a. 3 r. 29 p., correct to a square pole.

Square poles.

2,549(50·488 poles = 277·68 yards.
25

1004)4900
4016

884

The area is correct to 1 in 5000; the side can be, therefore,* obtained to 1 in 10,000, and may be stated as $277\cdot\frac{6}{7}\frac{6}{1}$ yards, or 277·7 yards.

* The *relative* error is doubled by squaring, and, conversely, halved in taking the square root. For, if x is a quantity subject to a small absolute error ex, $x(1 \pm e)$ will be the limits of the approximation to the value of x. Then $x^2(1 \pm e)^2$, which nearly equal $x^2(1 \pm 2e)$, since e^2 is small, will be the limits for the value of x^2, which is therefore subject to a relative error $2e$.

5. Multiplication, division and square root can be more rapidly performed by the use of logarithms, but there is considerable risk that part of the data will be lost, or a spurious accuracy introduced. If the data are correct to four figures, four-figure logarithms should be used, and the answer may be depended on to at least three figures, and similarly with other degrees of accuracy. Slide rules can also be used for special purposes, but their adequacy must be tested.

It is necessary to call attention to the complexity of these processes, because it is so commonly assumed that they are not worthy of attention. It is only a very competent arithmetician or experienced statistician who can see the effect of the inaccuracy of data throughout a problem. It is probable that many published statistics are less accurate than they appear, simply because the effect on the results of errors in the factors has not been considered.

It is to be observed that it is the most inaccurate of the factors or terms that governs the inaccuracy of the result.

6. Few statistical measurements are accurate to five figures, many not to more than three, and some are doubtful in the second figure. On the other hand, it is seldom that greater accuracy than 1 in 1,000 is required, and this can often be obtained.

It results that, in general, much space can be saved in tabulation and more accuracy be in reality obtained, by giving numbers only to three or four significant figures.

7. *Comparison and ratio.*—It is so much the custom to make comparisons by means of percentages, that the artificiality and, in some cases, the fallacy of the result are not perceived.

Suppose that we wish to compare two quantities, *e. g.* the aggregate values of Exports of Home Produce in 1898 (£294,014^{000}) and in 1907 (£517,977^{000}), and that we can depend on these values to four figures.

Any one of the following ratios expresses the facts—

$$2940:5180 = 1:1{\cdot}762 = {\cdot}5676:1 = 100:176{\cdot}2 = 1000:1762$$
$$= 56{\cdot}76:100 = 567{\cdot}6:1000 = 100 - 43{\cdot}24:100.$$

The ratio in italics is the simplest of these statements, if we take the value in 1898 as the standard of comparison, and that next written (·5676 : 1) if we take 1907 as the standard.

The statement most usually made would be (*a*) "The value has increased 76·2%"; it is more exact to say "The value in 1907 was 176·2% of that in 1898." The converse is "The value in 1898 was 56·76% of that in 1907"; the equivalent of this is (*b*) "The value in 1898 was 43·24% less than that in 1907." Few people would recognize that (*b*) was the converse of (*a*).*

After the phrase "per cent." the words "of x" are implied, where x is supposed to be known from the context. But the context does not always give definite information, as the following example of evidence given to a Royal Commission shows: "Wages were 15*s.* in 1870; they rose 20% between 1860 and 1870, and 10% more by 1875; by 1885 wages had fallen 25%." Any of the following would satisfy the statement—

1860.	1870.	1875.	1885.
12/6	15/-	16/6	12/4½, reckoning each period by itself.
12/6	15/-	16/3	13/1½, reckoning all on the 1860 basis.
12/6	15/-	16/3	12/2¼, reckoning the last on 1875.
12/4½	15/-	16/3½	13/0½, reckoning all on the 1885 basis.

From other evidence it appears that the third of these lines was intended.

One of the greatest strikes of the end of the nineteenth century was caused by a misunderstanding of this kind.

8. It would be an improvement in common methods if the decimal point were not used in comparisons; thus the statement as to exports would read: "the values are in the ratio 1000 to 1762." It would be a greater improvement if the ratio were always given, not the increase; thus "the value

* If x and y are two numbers, y is $100 \times \frac{y-x}{x}$ = say, u per cent. greater than x, and x is $100 \times \frac{y-x}{y}$ = say, v per cent. less than y. Then the simplest relation between u and v is $100\,(u-v) = uv$.

has changed in the ratio of 1000 to 1762," not " has increased 76·2 %."

Apart from the greater definiteness of the ratio statement we gain a further advantage in preserving the measure of accuracy. If average weekly wages change from 25*s.* 9*d.* to 27*s.* 3*d.*, each quantity being given correctly to the nearest 3*d.*, the *ratio* is between 25*s.* 10½*d.* : 27*s.* 1½*d.* and 25*s.* 7½ *d.*: 27*s.* 4½*d.*, *i.e.* between 1000 : 1048 and 1000 : 1068, or may be written 1000 : 1058 ± 10, and is known to 1 %. But the *increase* is only known as between 4·8 and 6·8 %, or as 5·8 ± 1·0 %, and is doubtful to the much greater extent of 1 part in 6. This source of inaccuracy is frequently ignored.

9. There are two groups of cases in which percentages (or per thousands, etc.) can be used without indefiniteness; they can be shown sufficiently by examples—

Value of Imports, received by the colonies, etc., from	00000's	Per cent. of total.	or	Per mille of total.
The United Kingdom .	£1434	46·4		464
British Possessions . .	561	18·1		181
Foreign Countries . .	1096	35·5		355
Total	£3091	100·0		1,000

In a long column of this sort, the percentage items, each calculated correct to the third figure, will not give in general 1,000 exactly as the total; the items should, nevertheless, be left as they are calculated.

(*b*) The second group is illustrated by the statements: " Per million males over 10 years of age in 1901 in England and Wales, 92,811 were occupied in *building and works of construction*, as compared with 34,898 per million in Ireland"; " Per thousand persons in England and Wales in 1871 and 1901, 437 and 470 respectively were between the ages 20 and 55."

Such methods of arranging numbers for comparison can hardly be distinguished from averaging, as dealt with in the next chapter.

10. The following examples illustrate common mistakes in the use of percentages—

"Of 57 persons, 35 (or 61·404 %) died." The number in the brackets is an example of spurious accuracy. In dealing with less than 100, the figure in the unit place is not established, and the decimals are absurd.

"Exports increased from £1,000 to £1,300, *i.e.* 30 %, but imports increased 500 %, the values being £20 and £120." Here are compared relative increases on values which are so different as not to be comparable; the *absolute* increase in the first case is three times that in the second. Such a statement is numerically correct, but is likely to be misquoted simply as "Exports increased 30 % and imports 500 %."

"Prices rose 20 % and then fell 20 %, returning to the former level." If the most natural meaning is given to the first clause, the three prices would be in the ratio 100 : 120 : 96 and the last price would be 4 % below the first. This kind of ambiguity and the resulting mistakes have already been discussed (p. 12).

"The total rose from about £143,000,000 to £185,473,000 an increase of 29·7 %." This should be "about 30 %."

CHAPTER III

AVERAGES

1. AVERAGES are of many kinds and have many uses. Here we deal only with the simpler averages and kindred quantities in common use, not involving mathematical analysis; and, avoiding formal definitions, we explain the methods and ideas by examples.

"1,000 cattle in the United Kingdom produce on the average 58 tons of meat per annum." We cannot say "1 cattle produces ·058 tons," for this is not true of an individual ox, cow, or calf; the use of the generic noun "cattle" itself suggests the more general statement.

An average of this kind is obtained by estimating the number of cattle and the amount of meat produced year by year over a period of years, and dividing the amount by the number.

The use of the statement is partly to abbreviate and to state in an accurate form (see last chapter) the result of a complicated investigation; partly to afford a basis by which the yield of the herds of the United Kingdom in future years can be estimated;* partly to make a standard of comparison with other countries and other dates.

2. In the census of 1901, 32,527,843 persons were enumerated in England and Wales, the area being 37,327,479 acres. There were, therefore, 0·871 persons per acre. In Worcestershire the "density" was 1·13, and in the county of London 60·62 persons per acre.

* The number of cattle is estimated by the Board of Agriculture every year; the quantity of meat is not estimated officially at all.—See *Statistical Journal*, 1909, p. 316.

This is an example of a fictitious average. To realize it, we have to make the absurd assumption that the persons are spread out over the country like butter on bread. Nevertheless, the statements in their most convenient form are of great importance for comparing the amount of land and of air space available in relation to the number of inhabitants, town by town and country district by country district.

Or, again, we may ascertain that land of certain qualities can support (say) three persons per acre on the average, and hence estimate the population that could obtain a living from a given district.

3. The population of the United Kingdom, June 30, 1907, is estimated to have been 44,099,000. The number of births registered for 1907 was 1,147,988; the birth-rate was, therefore, 26·0 (per thousand of the population per annum). Death-rates and marriage-rates are calculated in the same way. The use of these figures is for estimating the future population, for observing where the rates are abnormally high or low, so that, for example, sanitary measures may be taken with a view to reducing a high death-rate, and for studying the causes and effects of the fall in the birth- and death-rates which has been marked in recent years. These rates are averages of precisely the same nature as the yield of meat in the first example.

4. If the assessed annual value of the rateable property in a town is £900,000 and the common expenditure of the town is £300,000 per annum, a "rate" of 6*s.* 8*d.* in the £ would have to be imposed. Here the expenditure is averaged among the property-holders in proportion to the value of their property. In this case the average (expenditure ÷ assessed value) must be obtained first, and then the sum payable in respect of each property is calculated.

In 1910 the national expenditure of the United Kingdom was about £160,000,000, the population about 45,000,000; the aggregate of personal incomes was estimated as £2,000,000,000, but cannot be known within 10%. On these figures the necessary tax per head would be £3 11*s.* if all the money

were collected directly in equal amounts, person by person, and would be 1*s.* 5*d.* to 1*s.* 9*d.* in the £ if it were collected directly in proportion to income. By such averages an individual can estimate whether he is paying his due share of the national burden.*

The averages so far used are typical examples of arithmetical averages. An "arithmetical average" is usually defined as the quotient obtained by dividing the sum of several items by the number of items; this may be extended to include the quotient obtained by dividing a total by the number of persons or things connected with it.

5. If 25 lbs. of tea at 2*s.* are mixed with 50 lbs. at 1*s.* 6*d.*, the cost of the mixture is 1*s.* 8*d.* per lb. Conversely, if the prices of the constituents and the cost per lb. of the mixture were given, a simple arithmetic process shows that the proportions by weight of the constituents were as 1 to 2.

[Weight of dearer : weight of cheaper = Average − price of cheaper : price of dearer − average]

If 100 unskilled workmen at 25*s.* and 50 skilled at 37*s.* are employed, the average wage per workman is—

$$\frac{100 \times 25s. + 50 \times 37s.}{150} = 29s.$$

The last illustration is an example of a "weighted average," the numbers 100 and 50 being the weights in this case; the same process can, of course, be used for combining several groups.

A "weighted average" is obtained as follows:—Each of a series of quantities is multiplied by the number of persons or things connected with it, these multipliers being called "weights"; the sum of these products is taken as numerator, the sum of the weights as denominator; the fraction is the weighted average.

Examples and theory† show that slight errors in the

* Actually the problem is very difficult, since a great part is obtained in indirect taxation.

† *Elements of Statistics*, pp. 205–212 and 304.

"weights" have little effect on the average, if a fairly large number of terms are involved, none of them preponderant; and it frequently happens that the weights must be estimated, while the wages (or the other numbers concerned) are known accurately. Further, it is only necessary to know the *ratio* of the weights to each other, as a little consideration will show. If w_1, w_2, w_3 are weights, and n_1, n_2, n_3 numbers, the weighted average is $\frac{w_1n_1 + w_2n_2 + w_3n_3}{w_1 + w_2 + w_3}$. If the weights are changed, by multiplying each by k, to kw_1, kw_2, kw_3, the weighted average is $\frac{kw_1n_1 + kw_2n_2 + kw_3n_3}{kw_1 + kw_2 + kw_3}$, which clearly equals the former fraction.

One practical result of this principle is that the weights may be expressed in round numbers.

EXAMPLE.

Populations.	Numbers of Agricultural Labourers.	Weekly Wage.	Weights.	Weights.
	(1)		(2)	(3)
16,060	4,123	13*s.* 6*d.*	41·00	8
18,300	4,527	14*s.* 0*d.*	45	9
20,500	4,802	16*s.* 0*d.*	48	10
22,600	5,432	15*s.* 6*d.*	54	11

If weights (1) are taken, the average wage is found to be 14*s.* 10·0*d.*; if the round numbers (2) are used, the average is 14*s.* 10·1*d.* If it is observed that the numbers of labourers are nearly proportional to the populations, and if the weights (3), which are also nearly proportional to the populations, are used, the average is 14*s.* 10·3*d.*

The effect of taking approximate numbers for weights should always be carefully tested before the result is accepted.

6. In calculating averages of this kind, the work can often be greatly abbreviated without affecting its accuracy by either of the methods used in the following example. The proofs are left to the student.

CALCULATION OF THE AVERAGE WAGE OF THE GROUP WHOSE WAGES ARE SHOWN IN COLUMNS 1 AND 2.

1. Numbers.	2. Wages.	3. Wages 8s.	4. Product of Columns 1 and 3.	5. Wages 18s.	6. Product of Columns 1 and 5.	
27	8s.	+ 0s.	0	− 10s.	− 270	
23	10s.	2	46	− 8s.	184	
28	11s.	3	84	− 7s.	196	
41	12s.	4	164	− 6s.	246	
45	13s.	5	225	− 5s.	225	
49	14s.	6	294	− 4s.	196	
58	15s.	7	406	− 3s.	174	
61	16s.	8	488	− 2s.	122	
65	17s.	9	585	− 1s.	65	
65	18s.	10	650	0	—	
65	19s.	11	715	+ 1s.	—	+ 65
65	20s.	12	780	+ 2s.	—	130
62	21s.	13	806	+ 3s.	—	186
51	22s.	14	714	+ 4s.	—	204
48	23s.	15	720	+ 5s.	—	240
40	24s.	16	640	+ 6s.	—	240
33	25s.	17	561	+ 7s.	—	231
21	26s.	18	378	+ 8s.	—	168
16	27s.	19	304	+ 9s.	—	144
26	30s.*	22	572	+ 12s.	—	312
889			9132		− 1678	+ 1920 = 242

Using Column 4—

the average wage is $8s. + \frac{9132}{889}s. = 18s.$

Using Column 6—

the average wage is $18s. + \frac{2\ 2}{889}s. = 18s.$

Columns 3 and 5 are equivalent to Column 2. In 3, 8*s.* is taken simply because it is the minimum entry. In 5, inspection of the figures shows that the average is likely to be between 16*s.* and 20*s.*; 18*s.* was chosen as the starting point, as it appeared (without working) to be just below the average; the nearer the point chosen to the average, the less the numerical work required.

* Actually, "28*s.* or more."

The following table is a condensation of the one just given, and is suitable for rough, but fairly accurate, work.

Wages.	Numbers.	(a) Numbers.	Wages.	(b) In 5s. units.	Product of (a) and (b).
Below 10s.	27	3	say 7s.	7s. + 0	0
10s. and below 15s.	186	19	12s.	1	19
15s. „ „ 20s.	314	31	17s.	2	62
20s. „ „ 25s.	266	27	22s.	3	81
25s. „ „ 30s.	70	7	27s.	4	28
30s.	26	3	30s.	4·6	14
		90			204

$$\text{Average, } 7s. + \frac{204}{90} \text{ of } 5s. = 18s.\ 4d.$$

Here 12*s.*, 17*s.*, etc., are taken as the middle wages of the groups 10*s.* to 14*s.*, 15*s.* to 19*s.*, etc. If the wages were not in exact shillings, but were originally given as "23 persons earning 10*s.* and less than 11*s.*" etc., then 12*s.* 6*d.*, 17*s.* 6*d.*, etc., should be taken for the middle wage of the groups.

7. In distinction to the "arithmetical averages" described in paragraphs 1–5, which are mainly of use in facilitating further arithmetical processes, that in paragraph 6 may be called a descriptive average, for it can be used as an abbreviated way of describing the "group" of wages in the table.

The following sentences contain nine descriptive * averages. From the Board of Trade inquiry as to rents, prices and wages in the towns of the United Kingdom,† we learn that the average family weekly income was 36*s.* 10*d.*, the average number of children living at home was 3·6, the total expenditure on food was 22*s.* 6*d.*, of which 4*s.* 5½*d.* and 3*s.* 7*d.* were used for the purchase of 6·5 lbs. of meat and 32·0 lbs. of bread and flour respectively. The average rent for a five-roomed house outside London was about 6*s.*

* This word is not in general use as a technical term, but may be suggested as useful in classifying averages.

† Cd. 3864 of 1908.

Such averages are usually calculated by adding the total wages (expenditures, quantities, etc.) and dividing by the number of instances; that is, they are arithmetical averages, or (where the method of paragraph 5 has been used) weighted averages.

An alternative method of description would be to find out, *e.g.* the size of house which was *most commonly* used by the working-class; thus, if we know that 15, 25, 50, and 10 % of the families inhabited 3-, 4-, 5- and 6-roomed houses respectively, the 5-roomed house would be most usual or "predominant." We might further determine that (say) 6*s.* 6*d.* was the "predominant" rent. Our whole description might then be given in terms of "predominant" wages, rents, etc. As a sure description this is more vivid than the former; we should be describing the family of which, in fact, there were most instances, instead of an artificial family with 3·6 children. Such predominant rates are in statistics regarded as averages, and are technically called "modes" (fashionable, common).

The "mode" may be defined as that value of the graded quantity (wages, years, etc.) at which the instances are most numerous. Very generally in the statistics with which this book deals the apparent position of the mode depends on the accident of grading, and the mode cannot be exactly determined even by mathematical analysis.

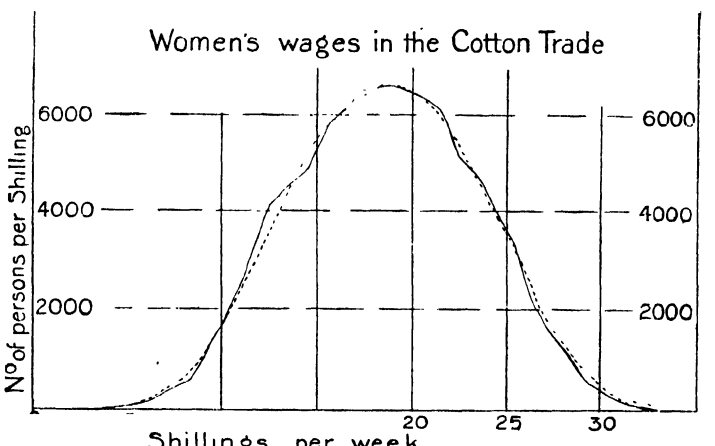
Another objection to its general use is that it is not obtained by a simple arithmetic process, and cannot be used, like arithmetical averages, for obtaining totals: if the arithmetical average of 3,000 men's wages is 30*s.*, the total wage is £4,500, but if we are told only that the "mode" is 30*s.* we cannot calculate the total.

The "mode" is more useful in anthropometrical and biological statistics, where there is a definite type, from which the measurements of the individuals of a group show deviations; in such cases the position of the mode affords precisely the measurement that defines the type.

Sometimes the word *average* is restricted to merely arithmetical measurements, while the word *mean* is used when a

group is described; if this distinction were made, "modes" and "medians" (see p. 24) would be means. But there is no general agreement on this point, and French and German writers do not make a corresponding distinction; we therefore regard the words as synonymous.

8. The group of wages given in paragraph 6 is a slightly modified statement of the weekly wages of women in the cotton industry. More complete figures are represented on the adjoining diagram. Such a diagram showing vertically



the relative numbers corresponding to the wages (ages, size or other measurements) marked on the horizontal scale is known as a "frequency curve," and a great part of more advanced statistics deals with such curves, which show the frequency of the occurrence of examples at various measurements.* Here we will only observe that the complete description of a group can only be given by such a curve or by an elaborate table, and that averages or means are only a shorthand or abbreviated way of describing some important characteristics of the group. The arithmetical average, which

* The dotted line in the diagram shows the effect of smoothing off the angles of the broken line; the latter represents the data as given.

shows on the horizontal scale the position of the centre of gravity of the area contained by the curve, and the "mode," which shows on the horizontal scale the position of the highest point, have already been discussed; in this case we certainly cannot obtain the latter correct to 1*d.*, as we can the former.

In paragraph 6 we assumed for simplicity that the wages were exactly at 10*s.*, at 11*s.*, etc. More accurately we now read the column as "10*s.* and under 11*s.*," "11*s.* and under 12*s.*," etc. Women's wages in the cotton trade are to a large extent piece-rates, calculated out to ½*d.*, and do not tend to arrive at exact shillings. The arithmetical average is in fact (as given in the Report, Cd. 4545, p. 28) 18*s.* 8*d.*, which we should have obtained if we had assumed that the average for such a group at "11*s.* and under 12*s.*" was 11*s.* 4½*d* and so on; actually some of the women are paid exact shillings, but many are paid by the piece and their earnings amount to any odd money; in the illustrative work we took it as 11*s.*, etc. No general rule can be given for such approximation; each case must be understood and judged on its merits.

9. Now make a new table from these figures as follows—

	Total (or cumulative) number.		Total (or cumulative) number.
Earning under	11*s.*	50	
	12*s.*	78	
	13*s.*	119	
	14*s.*	164	
	15*s.*	213	
	16*s.*	271	
	17*s.*	332	
	18*s.*	397	
	19*s.*	462	
	20*s.*	527	
Earning under	21*s.*	592	
	22*s.*	654	
	23*s.*	705	
	24*s.*	753	
	25*s.*	793	
	26*s.*	826	
	27*s.*	847	
	28*s.*	863	
	All	889	

Consider the values of *a*, *b*, *c* in the following statements; '*Half the wage-earners received a/- or less, one quarter received b/- or less, one quarter received c/- or more.*"

To determine *a* we want the position of the 445th worker (in order of wages from the beginning). The 397th worker just failed to reach 18*s.*; but 65 earned from 18*s.* to 19*s.*

and we need the 48th up this group. Making the not unreasonable assumption * that 65 were distributed uniformly 1*d.* by 1*d.* from 18*s.* to 19*s.*, we find that the 48th was at 18*s.* 9*d.*

The work may be shown as follows—

$$a/\text{-} = 18s. + \frac{\text{[illegible]}}{65} \text{ of } 1s. = 18s.\ 9d.,$$

$$\text{similarly, } b/\text{-} = 15s. + \frac{222 - 213}{271 - 213} \text{ of } 1s. = 15s.$$

$$\text{similarly, } c/\text{-} = 22s. + \frac{\text{[illegible]}}{705 \quad 654} \text{ of } 1s. = 22s\ 3d.$$

$a/\text{-}$ is called the "median," $b/\text{-}$ and $c/\text{-}$ are the lower and upper "quartiles" for this wage group.

The median and quartiles of a group may be thus defined: If the members of the group are ranked in order according to the measurement (wages, ages, height, etc.) under consideration, then the measurements of the members most nearly one quarter, one half and three quarters respectively along the rank are the "lower quartile," the "median" and the "upper quartile."

Such quantities obviously afford a very simple and definite description of a group. In fact, this method is the most helpful of the statistical abbreviations, and it is rapidly coming into common use.

The main objection to the median, as to the "mode," is that it does not lend itself to further numerical work. The following statement is true of the arithmetical average, but not necessarily of the median or mode—

If a_1, a_2 are the average wages of two groups of n_1, n_2 persons, then $\frac{n_1a_1 + n_2a_2}{n_1 + n_2}$ is the average for the combined group.

10. *Fallacies*—i. The average rate of a journey where alternate miles are done at 8 and 12 miles per hour, is not 10 miles

* Actually there is some concentration at 18*s.*; with full information this should be taken into account.

per hour, but 9·6 miles per hour, for two successive miles occupy $12\frac{1}{2}$ minutes.* The average rate of increase when three successive annual increments are 20 %, 30 % and 40 %, is not 30 %, but

$$\sqrt{\text{(}\times 1{\cdot}30 \times 1{\cdot}40\text{)} \times 100}\,\dagger - 100 = 29{\cdot}75\,\%.$$

ii. The average rate of interest of three sums of money bearing 3, 4 and 5 % respectively, is not necessarily $\frac{1}{3}(3 + 4 + 5) = 4\,\%$; *e. g.* if the sums are £1,000, £3,000 and £8,000 respectively, the interests are £30, £120 and £400, and the average rate is $4\frac{7}{12}\,\%$. "Weights" cannot be neglected without examination, nor unless certain special conditions are satisfied.

iii. If three groups of men have their wages raised each 20 %, the average is not necessarily also raised 20 % unless the relative numbers in the groups are unchanged. This is shown by the following example in which the average actually falls—

	At first Date.		At second Date (Wages increased 20%, but relative Nos. changed).	
	Numbers.	Wages.	Numbers.	Wages
Group 1.	100	20*s.*	400	24*s.*
Group 2.	200	25*s.*	200	30*s.*
Group 3.	400	40*s.*	100	48*s.*
	Total 700	Average $32\frac{9}{7}$*s.*	Total 700	Average $29\frac{1}{7}$*s.*

Neglect of a change of weights always distorts and sometimes reverses the results.

Again—

	Population A.		Population B.	
	Number.	Death-rate.	Number.	Death-rate
	44,000	16·4	44,000	16·2
Components:				
Under 5 years	4,000	25·5	1,000	26·0
Over 5 years .	40,000	15·5	43,000	16·0

Here the death-rate of Population A as a whole is higher than that of B, though the rates of the two parts shown are each lower; for A contains a larger proportion of young children, for whom the rate is high. ‡

* 9·6 is the harmonic mean between 8 and 12.

† The geometric mean.

‡ In connection with this example see the method of correcting the death-rate, p. 106, below.

In using arithmetic averages for the comparison of two groups, it is necessary to analyse the groups, and find if they are sufficiently homogeneous (of the same kind) in themselves to allow a reasonable comparison.

iv. *False accuracy.*—The average wage of two groups, the first of 100 men whose average is stated to be between 25*s.* and 26*s.*, the second of 200 men whose wage is between 30*s.* and 31*s.*, is not known to be

$$\frac{100 \times 25/6 + 200 \times 30/6}{300} = 28s.\ 10d.,$$

but is only known as between 28*s.* 4*d.* and 29*s.* 4*d.* Where there are many items, the average is more accurate than its constituents, but not necessarily when there are only two or three.

11. In an average the constituents of the numerator should be similar in kind to each other, and so should the constituents of the denominator. Also the various parts of the denominator should bear similar relations to the parts of the numerator. It is thus correct to speak of the death-rate of a population of healthy male adults, for they are subject to similar risks; it is correct to speak of the average wage of men in a trade. As we extend our view to include the whole population or a large group of trades, more and more caution is needed in the use of the average, though there are problems in which these wide averages are useful. It is doubtful whether any use can be made of the average frequently stated: "Total imports and exports divided by the population," as measuring the amount of foreign trade; for imports and exports are of different, even opposite, kinds for most practical purposes, and do not concern equally all the members of a population. Similarly "the average income per head of the population" can only be used for arithmetical purposes, not (except in a few cases) for comparison of one population with another.

CHAPTER IV

THE ACCURACY OF AVERAGING AND OTHER ARITHMETICAL PROCESSES

POPULATION OF THE COUNTY OF LONDON

	(1) Enumerated 1851	(2) Enumerated 1901	(3) Nearest 1000 1851	(4) Nearest 1000 1901	(5) Next 1000 under 1851	(6) Next 1000 under 1901
			000's		000's	
City of London .	127,869	26,923	128	27	127	26
Battersea . . .	10,560	168,907	11	169	10	168
Bermondsey . .	85,308	130,760	85	131	85	130
Bethnal Green .	90,193	129,680	90	130	90	129
Camberwell . .	54,667	259,339	55	259	54	259
Chelsea	54,078	73,842	54	74	54	73
Deptford . . .	24,899	110,398	25	110	24	110
Finsbury . . .	125,418	101,463	125	101	125	101
Fulham	11,886	137,289	12	137	11	137
Greenwich . . .	47,377	95,770	47	96	47	95
Hackney . . .	53,589	219,272	54	219	53	219
Hammersmith . .	17,760	112,239	18	112	17	112
Hampstead . . .	11,986	81,942	12	82	11	81
Holborn	95,676	59,405	96	59	95	59
Islington . . .	95,329	334,991	95	335	95	334
Kensington . . .	44,403	176,628	44	177	44	176
Lambeth	139,325	301,895	139	302	139	301
Lewisham . . .	18,616	127,495	19	127	18	127
Paddington . . .	48,415	143,976	48	144	48	143
Poplar	47,162	168,822	47	169	47	168
St. Marylebone .	157,696	133,301	158	133	157	133
St. Pancras . . .	166,956	235,317	167	235	166	235
Shoreditch . . .	109,257	118,637	109	119	109	118
Southwark . . .	152,371	206,180	152	206	152	206
Stepney	238,910	298,600	239	299	238	298
Stoke Newington .	6,076	51,247	6	51	6	51
Wandsworth . .	40,204	232,034	40	232	40	232
Westminster . .	244,178	183,011	244	183	244	183
Woolwich . . .	43,177	117,178	43	117	43	117
Total of the 29 districts	2,363,341	4,536,541	2,362	4,535	2,349	4,521
Averages. . . .	81,494	156,432	81,45⁰	156,38⁰	81,00⁰	155,9⁰⁰
Ratios 1851 to 1901	1000 : 1920		1000 : 1920		1000 : 1925	

1. The word error is used in statistics, not as meaning a mistake, but as denoting the difference between an estimate and the generally unknown exact measurement. We must distinguish between two methods of measuring error. In the adjoining table, the population of the county of London is shown as 4,536,541 in column (2), and estimated as 4,535,000 in column (4); the difference, 1,541, is called the *absolute* error; the ratio of 1,541 to 4,535,000, *i.e.* ·00034, is called the *relative* error. The *relative* error may also be expressed as a *percentage* error, in this case ·034 %. No simple rule can be assigned as to when absolute and when relative errors are the more important.

In the table, columns (1) and (2) give the populations of the city of London and the Metropolitan Boroughs in 1851 and 1901. Columns (3) and (4) give the same numbers to the *nearest* thousand; columns (5) and (6) give the same numbers *omitting* the last three figures in each case.

Example of *absolute* errors.—The average of the successive numbers 0 to 999 is 499·5. In numbers stated as in columns (5) and (6) we are equally likely to have omitted any number from 0 to 999, and are liable to an absolute error which cannot be greater than 29 × 999 or less than 0, and whose most probable value is 29 × 499·5 = 14,500 (nearly). The errors are actually 14,341 and 15,541, as may be seen from columns (1) and (2).

Example of *relative* errors.—The relative errors in column (6) are the ratio of numbers varying from 0 to 999, with average value very nearly 500, to the numbers in the column (26,000, 168,000, etc.). The smaller the population the greater the probable relative error. In the first line it is nearly $\frac{1}{26}$, while for Stepney it cannot be so great as $\frac{1}{2168}$. There is no simple relation between the relative errors in the items and in the total, except that the latter is between the greatest and least of the former.

2. It is clear that columns (5) and (6) under-estimate all the items and the total, while columns (3) and (4) are equally likely to be in excess and defect. Such errors as the latter

are called *fortuitous* or *unbiassed* errors, while the former (which all tend in the same direction) are *biassed.* The simple total of the absolute biassed errors in column (6) is the absolute error in the total. The case is very different for the unbiassed errors of columns (3) and (4). It is just as likely that they will be subtractive as additive ; actually 14 of the numbers in column (4) and 13 in column (3) are in excess, while 15 and 16 respectively are in defect. It is obvious that these errors possess a strong tendency to neutralize each other, but it is not obvious to what extent this neutralization will take place.

[Paragraphs 3 and 5 can be omitted without losing the sequence of the other paragraphs.]

3. The following rules must be accepted at present without proof, but they certainly appear plausible, and can be confirmed by experiment.

In the case of unbiassed errors—

(*a*) The *absolute* error in the total *increases* with the number of items, when each is subject to the same unbiassed absolute error.

(*b*) The best estimate for the absolute error in the total is the average absolute error to which the items are liable, multiplied by the square root of the number of items.

(*c*) The *relative* error in the total *diminishes* with the number of items.

(*d*) The best estimate for this relative error is the average absolute error of the items multiplied by the square root of the number of items and divided by the total.

(*e*) It is better to write (*d*) :—The best estimate for the relative error of the total is the average absolute error of the items divided by the average of the items, and also by the square root of the number of items.

Examples of (*a*) *and* (*c*).—If the first 4 lines only of column (4) are added, the absolute and relative errors are

respectively 730 and $\frac{1}{625}$, while those for the 29 lines are 1,541 and $\frac{1}{2944}$.

Examples of (*b*).—The average absolute error to which the items in col. 4 are liable is very nearly 250, all numbers from 0 to 500 being equally probable in the table above. The best estimate for the error of the sum (if we know nothing further about it) is $250\sqrt{29} = 1{,}346$, and the sum may be written 4,535,000 ± 1,346.* Actually column (2) shows that the true value is just outside this margin. The total for column (1) is just inside the similar margin (2,362,000 ± 1,346) obtained from column (3). We must not expect in general to be just at the margin.

Examples of (*d*) *and* (*e*).—The average of the items in column (4) is 156,400; their average absolute error is 250; their number 29. The relative error in the total is then estimated

from (*d*) as $\frac{250\sqrt{29}}{4{,}535{,}000}$, and from (*e*) as $\frac{250}{156{,}400\sqrt{29}}$, since 156,400 is the average item. Each of these = ·0003. The relative error found by comparing columns (2) and (4) is ·00034.

Similarly the computed relative error in column (3) is ·0006, and that found from columns (1) and (3) is ·0005.

Of course there is no means of determining what the error actually is when we only know the estimates. These rules only afford a means of estimating the errors to which we are liable.

4. The averages given in the last line but one of the table are of no importance except for illustrating the principles of this chapter.

It is evident that the absolute error of the average equals the absolute error of the total divided by the number of items, in this case 29.

It should also be evident that the relative error of he average is exactly equal to the relative error of the total.

* More exactly this means, "it is as likely as not that the total is within these limits, and very unlikely that it is as much as (say) six times as far from the estimate (4,535,000) as these limits are. The most probable value is 4,535,000, in the absence of information."

Biassed errors then remain in the average unaltered. The absolute error of the average will be very near the average absolute error of the items. Thus for both columns (5) and (6) the average errors may be expected to be (see paragraph 1) 500. We should therefore estimate the averages as 81,000 + 500 = 81,500 for 1851, and 155,900 + 500 = 156,400 for 1901, and these estimates differ very little from those shown in columns (1) and (2).

Unbiassed errors tend to disappear in the average just as they tend to disappear in the total. In fact, the absolute errors in the averages of columns (3) and (4) are only 44 and 52, and the relative errors ·0005 and ·00034 respectively.

5. The rules of paragraph 3 become for averages—

In the case of unbiassed errors—

(*b*) The best estimate for the absolute error of an average is the average absolute error of the items divided by the square root of their number; viz. $\frac{250}{\sqrt{29}} = 46$.

(*e*) The best estimate for the relative error of an average is the average absolute error of the items divided by the average of the items and also by the square root of their number, viz. $\frac{250}{156{,}400\sqrt{29}} = \cdot0003$ as before.

6. As a further illustration of biassed errors it may be noted that to obtain round numbers in a long addition of n items, we may carry $\cdot45n$ from the unit column to the tens, instead of doing the addition, since 4·5 is the average of the digits 0 to 9. From the hundreds column we may carry $\cdot5n$, since 50 is very nearly the average of the numbers 0 to 99. Similarly in adding money we may add $5\frac{1}{2}d. \times n$ for the pence, and $9s.\ 6d. \times n$ for the shillings, if the items end in pence and shillings respectively. If both pence and shillings are given we add $10s. \times n$ to the £.

Thus in column (1) by this rule the numbers to carry would be 4·5 × 29 = 13, ·5 × 29 = 14 or 15. Actually the numbers carried are 16, 16, 14, 15 in order.

7. *Comparison of similar totals or averages.*—Here we only deal with relative error. The actual ratio of growth shown in columns (1) and (2) is 1 : 1·920. That shown in columns (5) and (6) is 1 : 1·925. The relative error is $\frac{5}{1925} = \cdot0026$. The relative error is identical for averages and for totals.

The relative error of the ratio is very nearly equal to the difference between the relative errors of the two terms. If the errors are both positive or both negative, as is the case with biassed errors (unless there is a change of bias), the error in the ratio is less than that of the terms. Thus the relative errors for the totals of columns (5) and (6) are ·0061 and ·0032 respectively, both in defect; the difference is ·0029 very nearly the same as ·0026 just given.

There is no reason to expect that the small errors resulting from the addition of unbiassed errors will be both in excess or both in defect, though it happens to be the case in columns (3) and (4). In general we may expect the error resulting from unbiassed errors to be slightly greater in the ratio than in the terms.

The general result is that unbiassed errors tend to disappear in the averages and not to reappear in the ratio, while biassed errors tend to disappear in the ratio. The comparison of averages well constructed on similar principles generally has great accuracy, greater than that of the original items or totals. It has already been pointed out that the process of "weighting" also leads to accuracy. In fact, the ratio of weighted averages can under certain conditions which are often realized be obtained with a surprising accuracy. It can generally be determined by experimenting with the numbers whether these conditions are present.*

8. In dealing with a group, as in the last chapter, it is to be noticed that there may be a good deal of uncertainty about the extreme parts of the group, and yet the averages may be well determined. Thus the "mode" is not influenced

* For an example, see *Statistical Journal*, 1906, pp. 164 *seq.*

at all by anything except the central portion. The median is known completely for the table on p. 19, if the numbers (say) above 25*s.* and below 15*s.* are given, but not the exact wages in these marginal groups, and if numbers and wages are given in the central region; even if the top group, 26 at 28*s.* or more, were dropped out entirely, the median would only be lowered from 18*s.* 9*d.* to 18*s.* 7*d.* The arithmetical average is more easily affected by the position and magnitude of the extremes, especially the upper extreme; if of the 26 at 28*s.* or more (whose average in fact, is near 30*s.*), 6 were at 35*s.*, and 10 each at 40*s.* and at 50*s.*, the average would be raised from 18*s.* 8*d.* to 19*s.* 1*d.*; in such cases, general knowledge of the structure of the group will often make possible the assignment of narrow limits within which the average must lie.

9. When only two or three or a few terms are present, the rules given as to approximate work and round numbers in Chapter II apply. The greatest absolute error in the terms of an addition or subtraction, or in a factor of a product, dominates the error in the result. Many terms (say 20 or more) are necessary before the fortuitous errors can be confidently expected to neutralize each other. Of course, paragraph 7 above applies if the two terms form a ratio. The general practical rule in all cases involving few terms is to work through the problem, assuming every error is as great as possible under the conditions of the question, the sign of the errors being so chosen that they all work towards increasing the error in the result. Then give the answer in one of the forms of p. 7; if sufficient accuracy for practical purposes can be attained by giving the nearest round number which is certain, the statement "correct to the last digit given" is the best.

10. That a small absolute error in an item may have a great effect on the result may be illustrated by the following examples—

(*a*) Cost of workmen's budget.

	Prices.	
	1st date.	2nd date.
Meat, 8lbs. . . .	$8\frac{1}{2}d.$	$9\frac{1}{2}d.$
Bread, 20lbs. . . .	$2\frac{1}{2}d.$	$2\frac{1}{4}d.$
Total	9*s.* 10*d.*	10*s.* 1*d.*

Suppose that the price of bread had been obtained as an average 2·38*d.*, and had then been written to the nearest farthing, viz. as $2\frac{1}{2}d.$

Now suppose that a slight mistake had been made in the working of the average for this price, everything else being correct, and in fact it should have been 2·37*d.* Given to the nearest farthing this is $2\frac{1}{4}d.$ The first budget would then have amounted to 9*s.* 5*d.*, and the increase would have appeared as + 8 %. In this case a relative error of not more than 1 in 200 results in a relative error of 5 in 3.

A careful writer would have said in this case that there was no certainty of any change in the total.

(*b*) Of 695,720 members of Trade Unions, 7·4 % were unemployed at the end of September 1909. Seven groups of trades account for 579,899 members, of whom 8·5 % were unemployed. Can the number be deduced for the remaining group?

At first sight we might proceed as follows—

	Members.		Unemployed.
7·4 % of	695,720	=	51,483
8·5 % „	579,899	=	49,291
Residue	115,821		2,192

But the 7·4 and 8·5 are more exactly from the original figures 7·42 and 8·455, the total number unemployed was 51,749, and that for the seven groups 49,028. The residual number was therefore 2,721, which exceeds the estimate (2,192) by 25 %.

CHAPTER V

USE OF DIAGRAMS

1. DIAGRAMS do not add anything to the meaning of statistics, but when drawn and studied intelligently they bring to view the salient characteristics of groups and series they show the various parts in relation to each other and to the whole, bring to light the unity that underlies the scattered figures, and suggest in what directions investigation is needed. Merely pictorial diagrams are not only unlikely to be of much use, but in advertisements and political propaganda are often deliberately misleading, though literally correct. In the author's opinion the graphic method should rarely be used except (i) to show the relations of one part of a *group* to another (the word used in the sense of p. 1, where the various members differ in respect of one measurable characteristic), (ii) to exhibit a *series* of similar estimates date by date, (iii) to compare two or more groups, (iv) to compare two or more series, (v) to exhibit three relations which can be geometrically united.

Diagrams which simply show relative magnitudes—*e. g.* the populations of three countries at one date, or two isolated figures, such as the sale of some commodity at two dates, where the horizontal scale shows no graduated quantity (time, age, wage, height, etc.)—are of no assistance for the comprehension of the numbers.

Nevertheless, a skilful writer can often devise statistical diagrams of other kinds which help the visualization of a complex argument, and the aid received from diagrams varies greatly from person to person, so that it would be rash to lay down too rigid rules.

2. The four pages of diagrams given illustrate the main principles of graphic statistics and afford examples of most of the methods that are to be recommended. The first shows the two ways of representing a *group;* one is chosen that presents some difficulties, in order to show the elasticity of the method.

AGES OF MEN AND BOYS EMPLOYED IN COAL MINES, ENGLAND AND WALES, 1901.

Age.	Number.	Per Mille.		Cumulative
10-14 years	2,761	7	Under 14 years	7
14-15	3,992	10	15	17
15-20	36,469	89	20	106
20-25	67,349	164	25	270
25-35	131,818	322	35	592
35-45	86,735	212	45	804
45-55	53,305	130	55	934
55-65	22,073	54	65	988
65-75	4,645	11	75	999
75 and over	382	1	Total	1,000
	409,529	= 1,000		

The ordinary way of showing such a group graphically is that of B on the page opposite. The years are marked off on a horizontal scale. The numbers in the six equal age periods (15—25 years, 25—35 years, etc.) are represented by rectangles proportional to these numbers on any convenient vertical scale. It is customary, but inaccurate, to join the middle points (*a*, *b*, *c*, *d*, *e*, *f*) on the tops of these rectangles by straight lines, as in the figure. If there are many narrow rectangles, as in the diagram (p. 22) above, the inaccuracy is slight, and may be ignored.

In diagram B $\frac{1}{10}$th inch square represents 4 per 1,000 of the persons throughout. It is not difficult to see that if we represent the numbers at 15—20 years and 20—25 years separately, we must keep the same areal relation by doubling the vertical scale. Similarly, if the number at 14—15 years were shown, it would be represented by a vertical scale increased tenfold. This method will become clear as soon as an attempt is made to draw the diagram from the numbers.

Ages of Coal-miners.

Cumulative diagram showing the total number whose age is under each age marked on the horizontal scale:—
e.g. 804 (per 1000) shown as PN, are under 45 years.

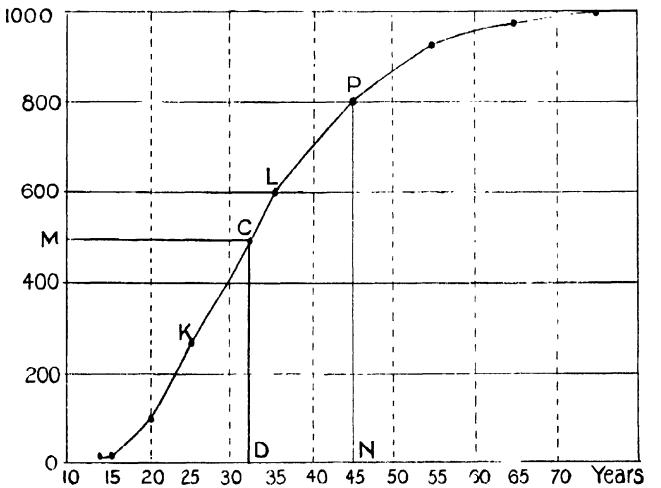
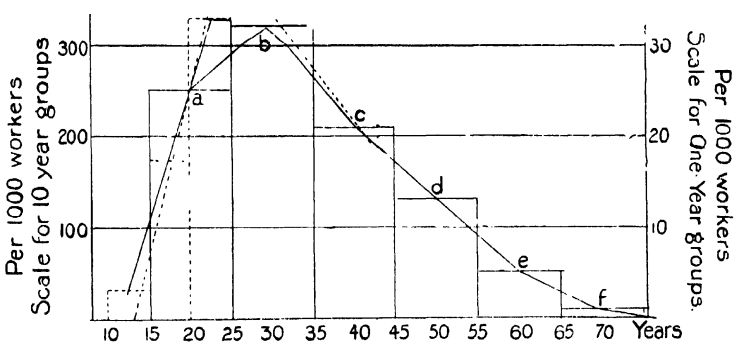


Diagram showing distribution by Age.



The rectangular blocks show the numbers in 10 year groups.
The dotted rectangles show the five-year groups from 10 to 25 years.
The curve shows the number at each year.

Since there can be no sharp division of numbers as we pass from age to age, the apparent division being introduced by the accidental placing of the age limits, it is clear that the whole group should be represented by an unbroken line. The ordinary introduction of such a line as *abcdef* is intended for this purpose. A little reflection will, however, show that we should keep the area standing on any given base unchanged, and that this line cuts off a great part of the area on 25—35 years. To avoid this a freehand curve should be carefully drawn, so as to keep all the areas unchanged. It will at once be seen that in this case the information is not sufficient for an accurate drawing, and that there is something arbitrary in the figure. If it is found that the line is not definitely placed, the figure should be left in the original rectangles.

Finally, the extremities of the curve, below 14 and above 70 years, must be drawn to satisfy the conditions of the data (in this case there are no children under 12 or 13 years), the continuity of the drawing being preserved.

The vertical scale adds very little to the information, and might in this case be removed after the drawing is complete with little loss.

These difficulties are present to some extent in all group diagrams.

3. Diagram A represents the cumulative numbers in the last column of the table, p. 36. The dots (K, L, P, etc.) show the information exactly as it is given, and there is no element of approximation or arbitrariness.

In the figure these dots are joined by straight lines. To obtain a more perfect representation the angles at K, L, etc., should be rounded off by a careful freehand curve, for there is no reason why the line should be broken exactly at 20, 25, 30 years, etc. The number up to any assigned age may then be read from the freehand curve. [To avoid confusion it is not drawn in the figure.]

It will be found that the curve cannot be finished at either end without further information.

The quartiles and median (see p. 24) of the group may readily be found approximately from the drawing. The line MC is drawn horizontally through the middle point of the vertical scale to meet the freehand curve at C; CD is then drawn vertically to meet the horizontal scale at D. The reading at D (32 years) is the median.

The mode is the reading above which such a curve is steepest, but is not easily determined with accuracy by the eye, and needs mathematical analysis before an exact value can be obtained.

A diagram of this kind is more accurate and useful than such as B, and is more easily used for the comparison of two groups. It requires practice to grasp its meaning readily.

4. The final test of the goodness of a diagram is its legibility and clearness of meaning.* The diagram should carry on its face a sufficient definition of the facts represented. There should never be many lines in one diagram, unless they can be kept apart from each other. Lines should be distinguished by colours or clear hachetting, and, where suitable their meaning (*e.g.* "weight" and "value," p. 44) should be, written close to them. Cross references should be avoided. If there is much detail, either the data should be separated into two or more diagrams, or the numbers should be left in a table and not represented graphically. An overloaded diagram defeats the only purpose for which it is intended.

Any diagram can be drawn on the back of a postage stamp or enlarged to cover a wall. The page of a book is generally sufficient for all the detail that ought to be shown, and large sheets and folded pages are to be avoided. The *ratio* of the vertical to the horizontal scale must be chosen so as to bring out those fluctuations or movements which are the subject of study; then the absolute scale should be so chosen that the allotted space is occupied.

5. The following diagrams show the method of representing

* Diagram B above is, of course, only intended to show the method of working. The other diagrams printed in this book satisfy, it is hoped, the conditions here laid down.

	Average Annual Gazette Price of Wheat per Quarter.	Quinquennial Averages.		Differences.
1864	40·2*s.*	—	—	
1865	41·8*s.*	—	—	
1866	49·9*s.*	1864–1868	52·0*s.*	− 2·1*s.*
1867	64·4*s.*	1865–1869	53·6*s.*	+ 10·8*s.*
1868	63·7*s.*	1866–1870	54·6*s.*	+ 9·1*s.*
1869	48·2*s.*	1867–1871	56·0*s.*	− 7·8*s.*
1870	46·8*s.*	1868–1872	54·5*s.*	− 7·7*s.*
1871	56·7*s.*	1869–1873	53·5*s.*	+ 3·2*s.*
1872	57·0*s.*	1870–1874	55·0*s.*	+ 2·0*s.*
1873	58·7*s.*	1871–1875	54·7*s.*	+ 4·0*s.*
1874	55·7*s.*	1872–1876	52·6*s.*	+ 3·1*s.*
1875	45·2*s.*	1873–1877	52·5*s.*	− 7·3*s.*
1876	46·2*s.*	1874–1878	50·0*s.*	− 3·8*s.*
1877	56·7*s.*	1875–1879	47·7*s.*	+ 9·0*s.*
1878	46·4*s.*	1876–1880	47·5*s.*	1·1*s.*
1879	43·8*s.*	1877–1881	47·3*s.*	− 3·5*s.*
1880	44·3*s.*	1878–1882	45·0*s.*	− 0·7*s.*
1881	45·3*s.*	1879–1883	44·0*s.*	+ 1·3*s.*
1882	45·1*s.*	1880–1884	42·4*s.*	+ 2·7*s.*
1883	41·6*s.*	1881–1885	40·1*s.*	+ 1·5*s.*
1884	35·7*s.*	1882–1886	37·2*s.*	− 1·5*s.*
1885	32·8*s.*	1883–1887	34·7*s.*	− 1·9*s.*
1886	31·0*s.*	1884–1888	32·8*s.*	− 1·8*s.*
1887	32·5*s.*	1885–1889	31·6*s.*	+ 0·9*s.*
1888	31·8*s.*	1886–1890	31·4*s.*	+ 0·4*s.*
1889	29·7*s.*	1887–1891	32·6*s.*	− 2·9*s.*
1890	31·9*s.*	1888–1892	32·1*s.*	− 0·2*s.*
1891	37·0*s.*	1889–1893	31·0*s.*	+ 6·0*s.*
1892	30·2*s.*	1890–1894	29·6*s.*	+ 0·6*s.*
1893	26·3*s.*	1891–1895	27·9*s.*	− 1·6*s.*
1894	22·8*s.*	1892–1896	25·7*s.*	− 2·9*s.*
1895	23·1*s.*	1893–1897	25·7*s.*	− 2·6*s.*
1896	26·2*s.*	1894–1898	27·2*s.*	− 1·0*s.*
1897	30·2*s.*	1895–1899	27·8*s.*	+ 2·4*s.*
1898	34·0*s.*	1896–1900	28·6*s.*	+ 5·4*s.*
1899	25·7*s.*	1897–1901	28·7*s.*	− 3·0*s.*
1900	26·9*s.*	1898–1902	28·3*s.*	− 1·4*s.*
1901	26·7*s.*	1899–1903	26·8*s.*	− 0·1*s.*
1902	28·1*s.*	1900–1904	27·3*s.*	+ 0·8*s.*
1903	26·7*s.*	1901–1905	27·9*s.*	− 1·2*s.*
1904	28·3*s.*	1902–1906	28·2*s.*	+ 0·1*s.*
1905	29·7*s.*	1903–1907	28·7*s.*	+ 1·0*s.*
1906	28·2*s.*	—	—	
1907	30·6*s.*	—	—	

a *series*. In a series we have generally to study both the short-period fluctuations (regular or irregular) and the general movement or tendency, or "trend," as it may be called. In Diagram A (p. 42) the jagged line shows the data as given. It is at once clear that we have a succession of rapid fluctuations combined with a general movement mainly downwards. The problem is to disentangle the "trend" from the fluctuations. The table on p. 40 shows how it may be done.

A period is selected, long enough to remove the fluctuations of separate years, short enough to allow a long series of averages to be obtained.* As in the second column of the table, averages are taken again and again, and the line of "moving average" is shown in the diagram. The angles and small fluctuations of this line should then be smoothed away, as they are accidental. This smoothed line shows the trend; in this case it is downward from about 1870 to about 1895, and nearly neutral with some inclination to rise in the most recent years. This line cannot begin at the beginning or end at the end, of the period covered by the data, for several years are necessary to establish "the trend."

It is now assumed that the smoothed line represents the course of the events, as determined by slow-acting, cumulative influences, and that the deviations from it are due to short-period (or, in some cases, accidental) causes. The deviations, or differences between the price of a particular year and the average price of the five years of which that year is the middle, are obtained in the table and represented in Diagram B; a new vertical scale is taken to throw the fluctuations into relief.

The smoothed line of Diagram A and the line of Diagram B show the "trend" and the "fluctuations"; but it is advisable to preserve also the jagged line of the first diagram.

* If the fluctuations occupy the same length of time (*e. g.* 10 years), again and again, this period (10 years) should be taken for the successive averages. It is not necessary to use the same period throughout the series.

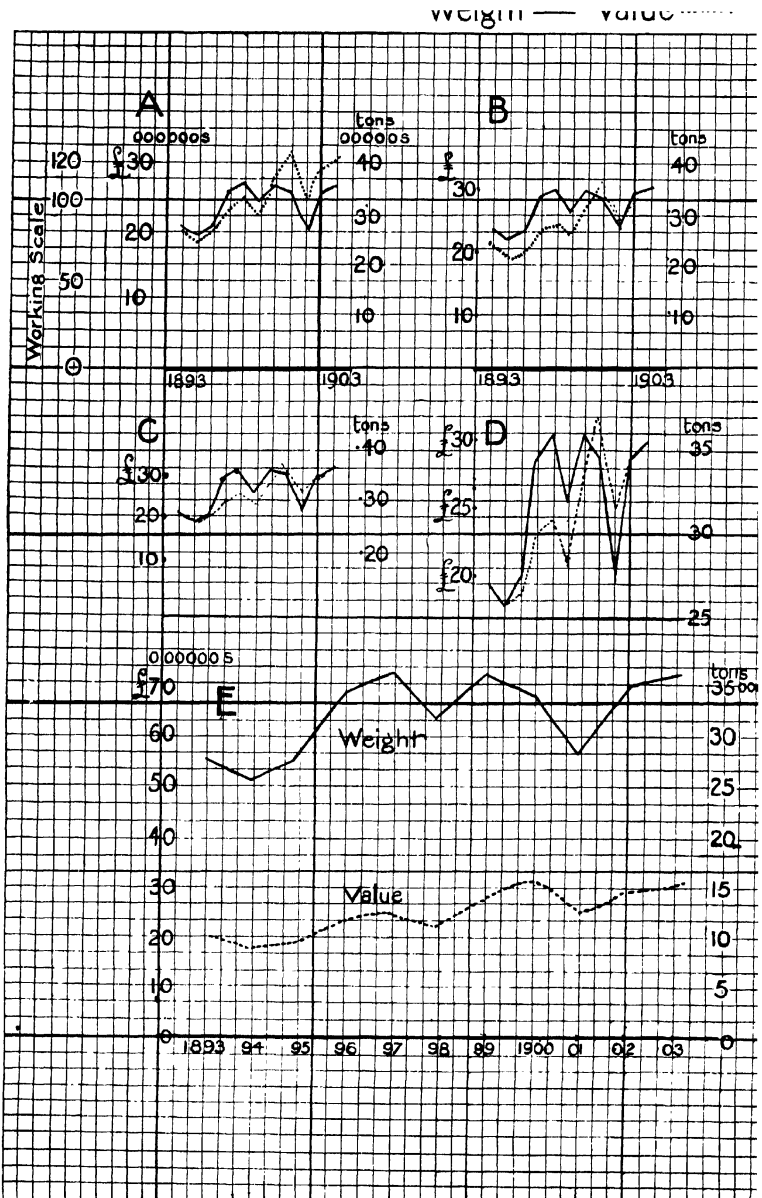
There is something arbitrary in Diagram B, since the magnitudes of the differences, and sometimes even their sign, depend on the length of the period taken for averaging.

6. Series can be distinguished by the nature of their "trends" and fluctuations; and it is extremely important to know both these with regard to any series used. The trends may be up or down, rapid or slow, uniform or changing. The fluctuations may be periodic (regular) or random (irregular), great or small. When we have examined the series, with the help of a diagram, over many years, we may know what to expect from the phenomena considered; we shall be able to tell whether a tendency observed is of a permanent character, and to distinguish between fluctuations which are natural to the series and those which show some great and new disturbance. For example, from this series we notice that the price fluctuates greatly from causes which may be present at any time, and that it would be quite impossible to trace the effect of (say) the introduction of some small new area into the world's supply, or the effect of a shilling import duty.

7. In order to show the relations of two terms of a series, or the size of the fluctuations relative to the total amount, it is essential to have a visible horizontal line through the zero of the vertical scale; otherwise continual and confusing reference must be made to the numbers on the vertical scale. This can be realized if the zero line of Diagram A opposite is covered up, and we ask if the price was halved between 1870 and 1890, or whether the fall from 1891 to 1894 diminished the price by one-third.

8. The next series of diagrams is designed to illustrate the danger of ignoring the zero line, some of the fallacies which an unscrupulous use of diagrams may render plausible, and the general method of comparing graphically two series. Figure E, on p. 44, shows the value and weight of iron and steel exports year by year on a scale which would naturally be adopted. There is no essential reason, however, why £10 and 5 tons should be represented by the same

Ⅳ Exports of Iron & Steel Manufactures



vertical distance. In the three figures A, B, C, the weight is represented on a uniform scale, viz. half that of E; but in A the scale for value is so chosen that the lines begin together, and also (as it happens) the averages for the eleven years of value and of weight are represented at very nearly

EXPORTS OF IRON AND STEEL AND MANUFACTURES THEREOF, PRODUCE OF UNITED KINGDOM

			Relative Numbers.		Relative Numbers.			
			A		B		C	
Years.	Value £ 0000's	Weight. Tons. 000's.	Value	Weight.	Value	Weight.	Value.	Weight.
1893	20,26	2,738	81	81	72	81	49·4 + 0	81 + 0
1894	18,47	2,566	74	76	65	76	− 4·3	− 5
1895	19,43	2,738	78	81	69	81	− 1·8	+ 0
1896	23,46	3,423	94	102	83	102	+ 8·0	+ 21
1897	24,41	3,599	98	107	86	107	+ 10·4	+ 26
1898	22,39	3,160	90	94	79	94	+ 5·1	+ 13
1899	27,71	3,601	111	107	98	107	+ 18·4	+ 26
1900	31,62	3,447	126	103	111	103	+ 27·4	+ 22
1901	25,01	2,813	100	84	88	84	+ 11·6	+ 3
1902	28,88	3,474	116	104	102	104	+ 21·4	+ 23
1903	30,40	3,565	122	106	106	106	+ 25·0	+ 25
Average	£24,73	3,193	99	95				

the same height. In B the equation is made for the last year, 1903.* Both these are correct, but method B very frequently gives the better perspective for two series. In long series it is best not to equate individual years, but to equate the averages of the last few years given.

C and D are misleading; the lines for value and weight are accurate separately, but the zeros of the vertical scales are not in the same position. It is a simple arithmetical

* To obtain the working figures for A, take 81 (a number convenient for numerical work in this case) to represent the value in 1893, and obtain proportionate numbers for the other years with a slide rule or otherwise. Take the same number to represent the weight in 1893, and finish the column by proportion. In this case easy arithmetic is obtained by multiplying the value by 4 and the weight by 3 less about 1 %. For B the same weight numbers are used, but the value in 1903 is equated to 106.

problem, of which part of the working is given in the table above,* to force the lines to begin and end together. D is merely C enlarged vertically.

A comparison of these five diagrams shows that almost any appearance may be given to fluctuations by a deliberate choice of scales, and suggests the need of care and intelligence in reading diagrams.

9. The following diagram shows one of the few methods of pictorial work that can be recommended. The proportion of the parts of a group to each other and the whole are shown by the sectors of a circle; since the areas of sectors are proportional to their angles at the centre and the arcs on which they stand, there is no possibility of confusing linear and areal proportions. For the comparison of two groups, two circles are constructed so that their areas are in the ratio of the numbers in the groups. It is at once clear by comparing the angles that the proportion (*e.g.*) of males between 14 and 15 years is smaller than that of

NUMBER AND AGES OF PERSONS OCCUPIED IN THE TEXTILE TRADES OF ENGLAND AND WALES (INCLUDING DEALERS), 1901

Ages.	Males.		Females.	
	Number.	Relative No.	Number.	Relative No.
10–14	24,700	18°	30,367	16°
14–15	18,332	13	31,402	17
15–20	81,200	59	188,125	102
20–45	267,168	196	359,976	196
45 and over	100,775	74	53,352	29
	492,175	360°	663,222	360°

$\pi r^2 = 4{\cdot}92175$; hence $r = 1{\cdot}252$ inches (1 sq. in. represents 100,000 persons),

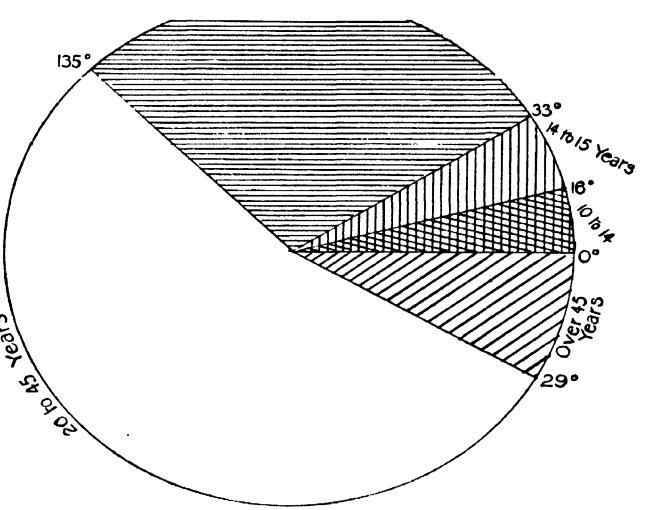
$\pi r^2 = 6{\cdot}63222$; hence $r = 1{\cdot}453$ inches (1 sq. in. represents 100,000 persons),

where r stands for the radius of the circle in each case.

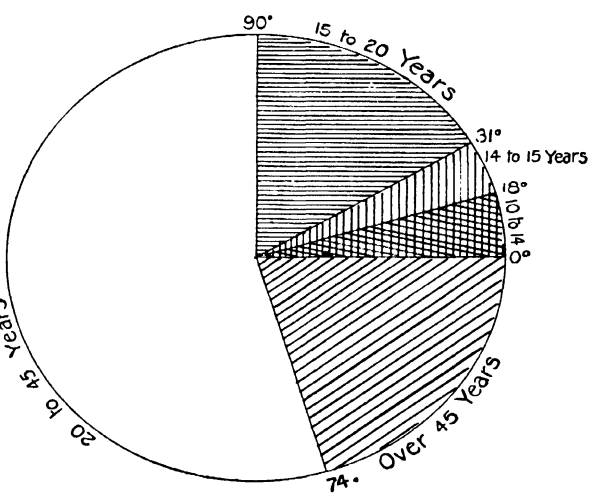
* The first and last figures for weight in A differ by 25. Equate the difference between the first and last figures for value (viz. 122 − 81 = 41) to 25, and reduce all the value numbers to the ratio 41 : 25; the first becomes 49·4, the last 74·4. Hence the numbers in the table.

V

Number & ages of Textile operatives



Females



Males

1 square inch represents 100,000 persons.

females; and observation of the areas suggests, (*e.g.*) that the number of women 20—45 years is about equal to that of all men over 20 years.

10. The commoner mistakes made in the construction and use of diagrams are as follow—

(*a*) By an injudicious choice of vertical scale the fluctuations are exaggerated (D, page 44), or, on the other hand, made inconspicuous (E, page 44, value line).

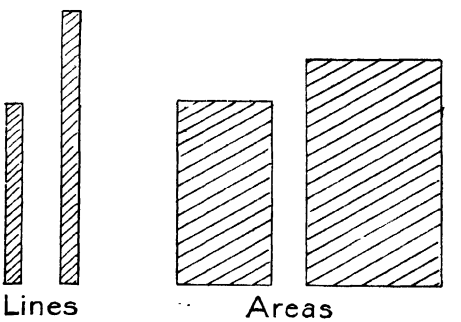
(*b*) An exaggerated vertical scale has the effect of making too conspicuous a single year in which the rise was greatest. It may easily happen that with monthly figures the high values would be seen to be spread over both the adjacent years.

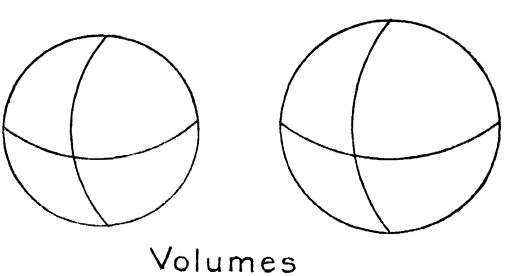
(*c*) When two series are represented on one diagram the equation is made between an exceptionally high year in one and an exceptionally low year in the other, with the result that the relative growths are distorted.

(*d*) It is not always realized that in such diagrams as B (p. 42) and E (p. 44), the dot representing the number is to be placed over the *centre* of the horizontal distance showing the corresponding period; while in A (p. 37) the dot is at the *end* of the period. Similarly the dots showing a moving average (A, p. 42) should be exactly at the centres of the periods for which the average is taken.

(*e*) In pictorial diagrams (such as the "big and little loaf") it is seldom clear whether the linear, areal, or cubic dimensions are intended to be compared. If one quantity is $1\frac{1}{2}$ times another, for linear comparison the ratios should be 1·5 : 1, for areal 1·225 : 1, and for volume 1·145 : 1. The three diagrams opposite illustrate the same ratio 2 : 3 in three ways.

VI Ratio. 2 : 3





CHAPTER VI

TABULATION

1. TABULATION is the intermediate process between the accumulation of data, in whatever form they are obtained, and the final reasoned account of the results shown by the statistics. The process of tabulation is essentially the selection from the data of all the persons or things, which have certain defined characteristics A, B, C, D, etc., and their subdivision according to other variable characteristics E_1, E_2, E_3, etc., and F_1, F_2, F_3, etc. Then ABCD (*e. g.* Cotton industry, weaving, men, 4 *looms*, in the table below) is the heading of the table; E_1, E_2, etc. (Ashton, Bolton, etc.), are the descriptions for the lines, F_1, F_2, etc. (under 20*s.*, 20*s.*–25*s.*, etc.), the headings of the columns. To any particular sub-group ABCD E_3 F_2 (4 *loom* men cotton-weavers at Stockport earning 20*s.* to 25*s.*) corresponds one entry (214) in the table. Of course the sub-divisions by the F's can be omitted for a simpler tabulation, or a third variable, G_1, G_2, can sometimes be introduced. In the table given 109 is the total of E_1, 799 the total of F_1.

It is advisable in many cases to tabulate in three successive stages: first, the ordered arrangement in full detail of all the information; second, the analysis of the first tables under definite headings as just described; third, abstract tables of the main results. The first set are merely for reference, if minute details may be wanted, or if further analysis may at some time be needed; the third set is a mere abbreviation of the second. In this chapter we deal with the second set.

2. The following table from the Reports on Earnings, etc., in the Textile Trades,* will serve to illustrate the discussion.

* Cd. 4545, p. 63.

Cotton Industry—Weaving

Number of Men Weavers (4 looms) working full time, whose Net Earnings in the last pay-week of September 1906 fell within the undermentioned limits.

Districts.	Under 20*s.*	20*s.* and under 25*s.*	25*s.* and under 30*s.*	30*s.* and under 35*s.*	35*s.* and above.	Total number.	Average earnings
							s. *d.*
Ashton-under-Lyne	9	67	29	4	—	*109*	23 10
Bolton	1	9	20	—	—	*30*	24 10
Stockport. . . .	10	214	75	—	—	*299*	23 3
Preston	37	406	361	23	6	*833*	24 10
Blackburn . . .	69	1,293	1,669	121	14	*3,166*	25 5
Accrington . . .	20	190	127	6	6	*349*	24 6
Burnley	185	1,448	1,942	402	86	*4,063*	25 11
Bacup	88	606	203	33	19	*949*	24 4
Rochdale	258	756	416	72	12	*1,514*	23 4
Other districts . .	122	535	147	6	1	*811*	22 7
TOTAL	*799*	*5,524*	*4,989*	*667*	*144*	*12,123*	24 11
Percentages . .	6·5	45·6	41·2	5·5	1·2	100	—

This is an example of double tabulation with cross totals. The problems isolated for study are the distribution of the number of weavers according to their earnings, and the variation of this distribution from district to district. It forms one of a series of tables in which the variation of wages according to occupation and district is examined.

3. Before making a table we must consider in detail exactly what information is wanted. The data generally consist of one or more items of information about each of many individual persons or things. In this case we know the industry, district, occupation, sex, age (whether adult or not), earnings, and length of time worked, for each person. We can group any three of these data in a double table. Here we take as the main heading the composite datum "industry, occupation, sex, age, and length of time (*i.e.* full time, 55½ hours)," and tabulate according to the remaining two, viz. district and earnings. We might equally well tabulate

district and occupation, or occupation and earnings, or sex and earnings, etc. The result of the particular tabulation used is to show that earnings are nearly uniform district by district, and are concentrated in the two groups 20*s.* to 25*s.*, 25*s.* to 30*s.*

Where one of the quantities varies grade by grade (as wages, age, etc.) it is entered in the horizontal heading. The number of grades entered separately is limited by the nature of the material and by the consideration that the whole must be easily visible at once.

The order of the districts, or other terms, in the vertical list should be alphabetical if there is no natural order; but it frequently happens that there is a natural or geographical grouping which is of assistance in studying the relationships, or in making subordinate totals. Similar places or things should be next each other.*

The line of totals shows the distribution of wages in the occupation as a whole; the column of totals shows the distribution of the occupation among the districts.

Supplementary information can be added, if the table is not overcrowded. A percentage line, as the last, is often very useful. The column of average earnings makes the visualization of the figures easier.

In printing, great care is necessary to bring out the principal words in the heading by suitable type; and wherever there is a change in the significance of the numbers a change of type should be made. A typewriter is not capable of producing a good table.

4. The table should, if possible, show on its face its exact meaning. It is too commonly the case that a table can only be understood by a cumbrous system of notes or references or by searching through a great deal of preliminary matter. For this reason the heading is rightly long and carefully worded. If necessary the heading should be broken up into

* In the table just given the order of places is that used throughout the report, and is convenient for cross reference. It is partly geographical, partly according to the nature of the trade in the district.

a series of sentences, with great care as to space and typing. When the matter in hand is extremely complicated, it is better to use a brief heading, and to place a full description of the meaning of the table and definitions of the terms used in print on the page opposite the table.

It often happens that many of the entries require special explanation. These may be given by a series of notes legibly printed immediately under the table. References by *, †, ‡, §, ¶, etc., should be avoided if possible. Every one has suffered from the system of notes used in railway time-tables. In the case before us, the only further definitions wanted are those of the districts and of the distinction between men and boys. The former is given by reference to a page where the delimitation of districts is stated once for all the tables; for the latter one has to search through the introduction to the report to find that males over 20 years of age are counted as men.

It is generally the case that, however minute the tabulation, there is a residuum; here we have "other districts," and earnings "35*s.* and above." The residua should be made small compared with the total, and should be inserted to avoid confusion.

After a table is made it is often the case that it has to be re-cast to fit the printed page. Folded tables should be avoided; if the table is too big for a page, or for two pages facing each other, it should be split up in two or more. The eye cannot grasp more detail at once than will cover two pages.

5. A table should neither contain numbers consisting of many digits nor many blank spaces. The latter can be avoided by merging the unimportant lines in the residuum. The former will be avoided if careful attention is paid to the substance of Chapter II above. Numbers have very seldom more than a superficial accuracy beyond the third or fourth significant figures, and it is seldom that greater accuracy is required, unless for further numerical work. Large numbers in a table confuse the eye, destroy the legibility of the whole

and conceal the significance of the grouping; the wood is hidden by the trees. Either round numbers should be used, in such a way that the last digit printed is accurate, or the lines can be given as percentages or per thousands. It is to be remembered that full details are supposed to exist for reference in an earlier series of tables (not necessarily printed).

6. So far we have been considering the form and nature of tables intended to give public information and resulting from a collection of statistical data. Tabulation has further important uses. When an investigation as to any facts is made, it may happen that the groups, or classes, or series which result are predetermined in form, and that we have merely to fill in details in tables already prepared; but it frequently happens that we are in the position of an explorer, and do not know even what kind of things we may discover. In such cases the process of tabulation is the process of analysis. In the investigation as to wages in the cotton industry, for example, tables were made to determine how far the number of looms tended per person influenced wages, what was the relation between the earnings of spinners and of their piecers, whether wages were nearly at a uniform level from place to place, and many other such questions. For analysis of this kind the rule is simple; determine exactly what it is that is to be tested, devise the table that will answer the particular question and no other, fill in the details from the data, and perform the necessary arithmetic for any comparison wanted

7. Again, in considering the progress of an institution or a business, analysis is constantly wanted, and is carried out in tabular form. We deal with this subject in Chapter IX.

8. Diagrams, averages and tabulation can all be used for presenting the results of a statistical accumulation. Of these the tabulation is the essential. Diagrams only give the results of tabulations in a special form, suitable for showing the relations between the various numbers and for allowing a *coup d'œil* over the whole field, but they cannot replace the actual figures for purposes requiring minute accuracy or for

further numerical work; also, as stated above, they should only be used over a limited field, while the tabular form is universal. Averages are abbreviations, replacing the more complete table for purposes of comparison with other tables. The reduction of a column of figures to an average throws away a great part of the data. Much attention has been given in recent times to curing this defect of averages, but after all refinements have been made we cannot dispense with the details of the group averaged, and these are to be found in tables.

9. It seemed inexpedient to load this manual with many examples of tables; in Part II many small tables are given, but they should be regarded as the final kind of tabulation, *i.e.* "abstract tables of the main results." The reader can find innumerable examples in statistical publications, and should criticize them by asking the questions: "Are the headings intelligible? Are the terms used in the heading and the table sufficiently defined? Is important information omitted or unimportant included? Is the spacing and arrangement of type satisfactory? Is there any difficulty in picking out the essential information?"

CHAPTER VII

SAMPLING

1. It is not always necessary to obtain complete information as to all members of a group, in order to give an adequate account of it. Most practical judgments are formed by experience of a limited number of examples. Purchases are frequently made after examination of a sample. The satisfactoriness of a consignment of goods is tested by examining and testing a few bars, cases, packages, etc. The probable yield of a mine is estimated by assaying a small quantity of ore. The goodness of a water supply is ascertained by bacteriological examination of a microscopic quantity. Such methods are not only means of saving time and expense, but are absolutely necessary in some cases; for testing often destroys the commodity, as when a tin is opened or the breaking-strain of a steel bar is determined, and it is often impracticable to examine every part, *e. g.* in the case of a mine, whose contents is not completely known till it is exhausted.

2. The first essential of an examination by sample is that every member of the group considered should have nearly the same chance of being included in the sample. This may be secured either by mixture or by random selection. *Mixture.*—Suppose it to be required to assay the quantity of gold in several barrels of the sweepings of the Mint, or the quantity of alcohol in many cases of wine, to take two eminently practical examples. In the first case, extract equal small quantities of dust from near the top, the middle and the bottom of each barrel. Mix each sample thoroughly, take an equal fraction of each and mix (say) four together; repeat this process of mixing and division till a quantity small enough to be assayed is obtained. In all such processes the methods of choice, mixing and division will be directed to neutralizing any physical irregularities of weight, shape, etc., which might

destroy the random nature of selection. To determine how nearly the result is correct, the process should be repeated (say) four times; the true result may be expected to be within the divergencies shown by the four measurements.

3. *Random selection.*—This is often sufficiently secured by the process of spreading out the consignment of goods, etc., and marking one taken here and another there, avoiding the first and the last and the most obvious, and testing the objects marked. Another method is to divide the objects into equal groups and take one at random from each group. The more scientific way is to secure absolutely equal chances by numbering the whole group consecutively, writing down the numbers on tickets and shuffling them, and finally drawing at random some of the tickets and examining the objects with the corresponding numbers. To avoid the writing and drawing, digits are sometimes selected at random from mathematical tables and used as if they were numbers drawn at random.

As before, the exactness of the result (if it is a case of measurement) should be tested by repeating the process, varying the selection each time.

4. In carrying out the above processes successfully in social or other investigation, less concrete than the examination of a consignment of goods, the first step is the careful and exact definition of the group to be tested. If, for example, we are examining the physical condition of school children, we should delimitate the area to be taken, enumerate all the schools in it, and find the number of children on the register of each; the group taken would then be co-extensive with the "registered school children." In making the measurements we should have to take children absent from school as well as present, if they happen to be chosen by the selective process used, as otherwise we should be taking the smaller group "children present at school"; this might give an imperfect result, as the absent children might contain a large proportion of the physically unfit. In any case, the group as described would not contain children removed from the district and specially treated in institutions.

The temptation is always to measure the obvious and

easily accessible; but if we do this our sample is of "the accessible," not of the whole group. Thus the budgets of working-class expenditure, which are often published, are not typical of the working class as a whole, but of that part of it which is intelligent enough to have some kind of record and is willing to communicate private details. In particular, the expenditure on drink is under-estimated.

5. *Determining the average.*—It is clear from common-sense principles that the larger the number included in the sample measurement, other things being equal, the more accurately the average will be determined; in Chapter IV it was stated that the precision increased as the square root of the number taken. This accuracy does not depend in any way on the *size of the group* from which the sample is selected; the average height of all the men in England can be determined with the same accuracy by the same number of measurements as the average in one town, if in each case every person has the same chance of inclusion. The following examples illustrate the increase of precision as more samples are included, and other points—

(*a*) Forty groups of ten entries each were taken at random from a list of the rate of interest paid by 3,878 Companies.

The average rates obtained for these forty groups were as follows—

AVERAGES OF 10 COMPANIES SELECTED AT RANDOM

Rate of Dividend.	Number of Occurrences.
£ s. d.	
Above 5 0 0	1
4 18 6	3
4 17 0	5
4 15 6	7
4 14 0	6
4 13 0	8
4 11 6	7
4 10 0	3

The average of the 400 Companies, contained in the 40 groups, is £4 14*s.* 11*d.*

The original entries vary from 0 to £103 %. The averages of 10 are all between £4 10*s.* and £5 1*s.* It is then practically certain that the average of all is between these limits, and not far from the average of the 40 groups, viz.

£4 14*s.* 11*d.** Actually it is found to be £4 15*s.* 7*d.*, when all the Companies are included.

(*b*) A large number of packs of playing cards were mixed together, and 32 groups of 3 cards were drawn, and the number of pips on each were counted, Knave, Queen, King being taken as 0. The following was the result—

Total number of pips on 3 cards in order of drawing.	Total on 12 cards.	Average per card.	Total on 24 cards.	Average per card.	Total on 48 cards.	Average per card.	Total and average for 96 cards.
5 16 17 18	56	4·7					
14 19 24 8	65	5·4	121	5·0			
8 5 8 19	40	3·3			206	4·29	
9 10 18 8	45	3·75	85	3·5			402
0 11 15 9	35	2·9					Average 4·19
17 18 14 16	65	5·4	100	4·2			
10 7 7 22	46	3·8			196	4·08	
22 17 5 6	50	4·2	96	4·0			

* These figures are given and more refined measurements are made in the *Statistical Journal*, 1906, pp. 550–3.

The original cards vary from 0 to 10; the averages of 3 from 0 to 8, of 12 from 2·9 to 5·4, of 24 from 3·5 to 5·0. It is then practically certain from the sample that the average of all is between (say) 3·5 and 5·0, and that 4·19 is a good approximation. Actually there are 55 pips to a suit of 13 cards (picture-cards counting blank), and the average is 4·23.

6. While the determination of the average is of great practical importance for purposes of valuing the group and other arithmetical work, it is often equally important to determine the proportion of various kinds in a group, as for example the number of families per 1,000 whose income is less than £1 per week, or the number of children per 1,000 suffering from remedial throat complaints. The following examples show a method that can be followed—

(*a*) The 400 Companies in the former example were divided into 4 groups of 100 each and tabulated according to the rate of dividend paid.

Rate of Dividend.	Number of Companies. 1st 100	2nd 100	3rd 100	4th 100	Together.	Per cent. Estimate.	Per cent. Actual.
Nil	6	5	8	9	28	7	6·0
£1 and under 3	3	0	3	0	6	1·5	1·5
£3 „ 4	34	23	29	22	108	27	27·2
£4 „ 5	25	30	28	34	117	29·25	31·1
£5 „ 6	13	18	16	13	60	15	17·7
£6 „ 8	9	16	9	14	48	12	10·8
£8 and above	10	8	7	8	33	7·25	5·7
	100	100	100	100	400	100	100

The last column but one shows the distribution as estimated from the sample of 400; the first 4 columns show how far the estimate can be trusted. Thus it is practically certain that rather more than half the Companies paid between £3 and £5, the numbers only varying in the 4 groups from 53 to 59. The number between £1 and £3 is doubtful. The last line, containing the exceptionally high dividends, is *a priori* uncertain; the accident of sampling may easily include too

many or too few rare cases. The method can only be trusted for the large, central divisions. The last column shows the actual distribution of the 3,878 Companies, from which the samples were taken.*

(*b*) In the draw of 91 cards (including all but the last five of the previous paragraph), the actual occurrence of the various numbers was—

Ace	8	
2	8	20
3	5	
6	8	23
7	8	
8	4	
9	9	19
10	6	
Knave	7	
Queen	7	21
King	7	

If the drawing had been continued, of course, we should have found less and less relative difference between the numbers. Here we have no actual test of the accuracy of the result.

[A mathematical way of dealing with such a question as "how many picture-cards are there in the given group?" is as follows: Let n be the number in the group, let m be drawn, and pm prove to be pictures. Then pn is the most probable number of pictures in the pack, but it is as likely as not to differ from this number by as much as $\frac{2n}{3}.\sqrt{\frac{p(1-p)}{m}}$. It is very unlikely to differ by as much as six times the expression just written.

In the card experiment, if n were 1 820 (the number of cards actually in the group used), $m = 91$, $pm = 21$. The

* For the *a priori* test of accuracy, see again *Stat. Journal*, 1906, p. 553.

forecast from the sample as to the number of pictures among the 1,820 cards would be—

and we should feel sure that the number was between $420 \pm 6 \times 54 = 420 \pm 324$.

Per 13 cards we should have (by proportion) $3 \pm \cdot 4$,* and the maximum possible would be about 5.

Thus the experiment is not sufficient to determinate the proportion of picture-cards accurately. More cards would need to be drawn till the m in the above formula was sufficiently increased.]

7. No formal rules can replace judgment and experience in the selection and interpretation of samples. The simplest practical direction is to continue to increase the number of samples, till successive tests show sufficiently similar results. When dealing frequently with the same kind, of course, experience would soon show how many tests were sufficient.

8. Two other methods of sample measurement are sometimes used.

Suppose we wish to test the knowledge of a large class of students (say 100). We might by some very simple examination, or by consulting the teacher, place them roughly in order of intelligence, and then examine in detail, say Nos. 1, 10, 25, 50, 75, 90, 100 (the maximum and minimum, median, quartiles and two deciles).† Thus a good estimate could quickly be obtained, and the relative ability of two similar classes quickly judged.

In the same way we could describe any group that can be placed in order, by the detailed examination of a few *selected by rule*. This method differs essentially from the method of random selection already explained.

* Observe that this result is independent of n.

† The deciles are the values which divide a group into ten equal parts, in the same way as the quartiles divide it into 4. If these seven positions are determined quantitatively for any group, a diagram of the form A, p. 37, can be drawn with considerable accuracy.

9. Rather than trust to the arbitrary action of chance, some investigators prefer to choose what they believe to be typical groups, and examine them in detail. Thus, investigations as to the wages, etc., of agricultural labourers have been conducted by selecting some forty districts throughout the country, so as to include types of all kinds of agriculture, and of all economic situations. This method results in an accurate and intelligible picture, but there is no easy means of calculating any average, or of knowing the distribution by number of persons earning various rates of wages. For filling in details where the general results are known the method is to be recommended.

CHAPTER VIII

RULES FOR USING PUBLISHED STATISTICS

1. It is never safe to take published statistics at their face value, without knowing their meaning and limitations, and it is always necessary to criticize arguments that are based on them, unless one is able to trust implicitly the knowledge and good faith of the persons bringing them forward. It is extremely easy to falsify the lessons which numerical statements should teach. The actual use and appreciation of statistics is ultimately a matter of intelligence, special knowledge and common-sense; but the following nine rules suggest the lines of study and criticism.

First.—Find the exact definition of the units which go to make the total. What is a soldier? What one pound's worth of exports? What a registered birth? What a member of the population, a case of fever, a bushel of wheat? One of the standard questions in agricultural statistics is "What is a cow?" In every case the definition depends on the regulations and method of collection. Thus we need to know at what stage a recruit is entered as "on the strength of the regiment"; what goods are counted as exports, and how they are valued; whether all births are registered, and whether still-births are included; how travellers, absentees and the homeless are counted; what are the rules for diagnosis of fever; whether wheat is weighed or measured; when a heifer grows into a cow, and much more detail of this sort. Generally expert knowledge is needed; sometimes the report on the statistics contains sufficient explanation and definition; sometimes the whole can be worked out from a study of the blank forms of

inquiry (with instructions) on which the original data are obtained.

The apparent meaning of a total is seldom its real meaning, but generally results from an artificial definition, necessitated by the process of collection.

As examples may be suggested the discovery of what is meant (i) by a room, (ii) by a farmer, in the census reports.

2. *Second.*—Consider how far the persons or things grouped together in a total or sub-total are similar; in other words, how far the group is homogeneous. Thus, persons whose occupations are grouped under the main heading "Textile Fabrics" differ with respect to (1) sex, (2) age, (3) nature of the material worked (cotton, wool, etc.), (4) position in the industry as merchant, dealer, manufacturer, or employé, (5) specific occupation, (6) locality. If we are merely told that 1,155,397 persons were included under the main heading in England and Wales in 1901, the information is so wide as to be nearly useless. An example of the most minutely defined group given in the census reports is:—County Borough of Oldham: number of males between the ages of 25 and 45 engaged in spinning process in cotton was 2,711. To know the meaning of this we should have to go carefully through a spinning-mill.

Whether the group or sub-group is sufficiently homogeneous depends entirely on the purpose for which the figures are used. If we compare the total numbers in the cotton industry in 1891 and 1901 we should be misled, because the numbers of children, men and women are in quite different proportions at the two dates; but a useful comparison might be made between the numbers of men.

The possibility of change in the relations shown when the groups are analyzed into parts of greater homogeneity must always be borne in mind. Innumerable examples might be given; an important one arises from death-rates. The rate is calculated by dividing the number of deaths in a district in a year by the number of persons living in the district

midway through the year, and multiplying by 1,000. Analysis at once shows that the various age-groups of the population are subject to quite different risks of death, and that the risks differ also according to sex; further, deaths from accident, from infectious diseases and from other causes should be in different categories.

If the internal constitutions of two groups are the same, e.g. if the distribution by age and sex are the same, then averages based on them may be properly compared. But we must never assume either homogeneity or similarity of division without knowledge.

3. *Third.*—Having defined and analyzed the totals, the next question is what is the relation of the quantity they measure to the quantity as to which we want knowledge. We wish to know the stress of unemployment, we learn the number of trade-unionists out of work; or of poverty, and we are told the number in receipt of public relief; or we are examining the improvement in health of the population, and we find the amount of disease and the number of deaths; for education we can tell the number of students, or of student-hours, or of examination successes. These statistical totals and averages are at best indices ,not actual measurements, of the more subtle and often incommensurable quantity or quality, which is essentially the object of the investigation. In order that indices may be useful they must at least move up and down with the quantity they represent, as the thermometer moves with heat and the barometer with pressure, and they should further make great or small oscillations with great or small movements; but many of them have less relation to the complete phenomena than the thermometer has to sensation of heat (which depends also on moisture and physiological conditions), and may be as remotely connected as the fall of the index of a barometer with the fall of rain.

If experience shows that the indices are sensitive and trustworthy they may be used to bridge over the gap between one more complete measurement and the next.

4. *Fourth.*—Before trusting or even reading a statistical

account, it is well to sit down and think quietly what statistics ought to have been collected, if possible, for the purpose in hand, and what sources of information exist, or should exist. Thus, if wages were to be measured, we should decide that the weekly rate, the annual earnings allowing for unemployment, supplementary earnings and the earnings of other members of a man's family should be known, and that allowance should be made for any necessary expenses; further, the money value should be interpreted in purchasing power, and the standard of life attained should be clearly shown. Of these things some it is not possible to measure; we cannot measure the actual satisfaction obtained from the expenditure of money, nor the value of unpaid personal work. Others, as the annual receipts and complete expenditure, could only be measured if the persons concerned kept accurate accounts.

Having got so far, we may take up the statistical report and consider how far the problem has been understood, whether all the practicable measurements have been made, and whether the result gives a true index in the sense of the last paragraph. We can thus decide as to whether the information is sufficient for solving any assigned problem; in only too many cases we find that it is not.

Further, if there is any suspicion of bias, of the intention to support any preconceived view, the criticism of method must be particularly rigid, and the maximum possible effect of the unconsidered factors must be allowed for.

5. *Fifth.*—When we have to deal with averages, rates, and percentages, we must carry our second rule of criticism further. Not only must we consider whether the numerators and denominators are homogeneous in themselves, but whether the terms of the denominator have a reasonable relation to those of the numerator. Should, for example, the number of deaths from small-pox be counted in relation to the whole population, to the vaccinated population, or to the number who contract the disease? Should the birth-rate be reckoned per 1,000 of the population or per 1,000 of married couples? Should the production of coal per head be reckoned

with respect to the population of a nation, or to those engaged in the coal trade, or only to the coal-hewers? The general answer is that the denominator should be limited to those who have a direct relation to the numerator; the legitimate birth-rate, *e.g.* should be in relation to married couples with some restriction of age. It may happen that this restricted denominator has a constant relation to a larger population, and in that case the latter may be used for simplicity of working (sometimes for lack of the detailed information), and for comparison with similar averages. Thus the number of births (929,807) in 1901 in England and Wales may be stated as 160 per 1,000 married women or as 28·3 per 1,000 persons; this last results from the combination of the two rates, 160 births per 1,000 married women and 177 married women per 1,000 persons. If the 177 remained unchanged, the two rates 160 and 28·5 would of course have a constant ratio to each other.

6. *Sixth.*—When two quantities are compared we must consider whether they are strictly comparable, and for this purpose most of the foregoing rules are necessary. Comparisons are made between two similar measurements at different dates (*e. g.* population, death-rate, average wage, production of wheat, etc.), or between two similar measurements relating to different places (*e. g.* trade, consumption of meat or wheat per head, amount of taxation per head, total or average income in two countries, etc.). We must test whether the two measurements are made on the same basis, so as to be indices of the same kind of phenomena considered, so as to cover the same ground and suffer from similar "error of bias" (see pp. 29, 32). Having ascertained this and so used rules 1 and 3, we then apply rules 2, 4, and 5 if necessary.

By such means we shall readily realize the difficulty of minute comparisons over long periods, during which relations have continually changed, and the extreme roughness of comparisons between such measurements as the indices of prosperity of two nations. Accurate comparisons can only be made between closely similar things or over quite short periods.

7. *Seventh.*—Closely connected with the last is the measurement of accuracy. In Chapters II and IV the approximate nature of statistical measurement was discussed, and some methods were given of testing the accuracy of results. In all statistics we must decide whether the data and methods will yield results accurate enough for the arguments based on them. It would be absurd to speak of an increase in average wages from 20*s.* 3*d.* to 20*s.* 6*d.* in twelve years, for the average could not be determined to 1*d.* in either case, and the group considered would have changed its character in the period; but we could speak reasonably of an increase of "about 50 %" if the averages were 20*s.* and 30*s.* The less the groups satisfy the stringent conditions of the first six rules laid down, the greater must be the margin allowed for error. Where possible, the greatest possible errors arising from imperfection of data or processes should be worked out.

8. *Eighth.*—We must not depend on figures relating to single days, months, or years, or on comparisons relating to short isolated periods. In Chapter V the fact that every measurable recurring phenomenon yields a series of definite characteristics was illustrated. These characteristics, the natures of the fluctuations and of the trend, must be known In the case of the population of a large country, where there is little emigration or immigration, it is not difficult to fill in estimates for intermediate years; in the case of the total value of exported goods it is impossible. Every measurement must be viewed in the light given by a series of similar measurements stretching back over a long period; otherwise temporary fluctuations will be taken for permanent changes, as if a cold summer were regarded as proving a change in climate; or a rise will be reported, when the whole trend is downwards, as if we should compare the bank holiday traffic of a decadent tramway one year with the lowest day's record of this preceding.

Where a sufficient record cannot be obtained, judgment must be suspended.

9. *Ninth.*—Having determined as far as possible the exact

purport and limitations of the statistics, consider (without reference to the printed report) to what conclusions they lead, or whether they are so imperfect that no conclusions can be reached without further investigation. There is often a great gap between the statistical table and the non-statistical conclusions that are fathered on to it, especially if the statistics were obtained in order to support a preconceived theory. Statistical work properly ends with such a dull, colourless, matter-of-fact report as is customary in the publications of the British Government. As a separate process such results are to be taken in conjunction with non-statistical knowledge. Inferences are suggested and tested by the reported facts, and a severely critical and logical analysis is necessary before the whole investigation leads on to some reasoned action.

CHAPTER IX

METHODS OF STATISTICAL ANALYSIS

1. In the previous chapter the way to criticize statistical reports was outlined; in this chapter we consider briefly the methods of collecting statistics at first hand, (i) for the purpose of testing the progress of a commercial undertaking, (ii) for testing the success of an institution, (iii) for collecting data for the solution of a social problem.

2. Details vary so greatly for different kinds of business, that it is only possible to lay down some general principles with illustrations. The processes of book-keeping and accountancy are, in their more refined forms, examples of statistical investigation, and, so far as £ *s.* *d.* is concerned, provide the data, even if they do not give the result, of such analysis. When accountancy is applied to commodities as well as to money, we arrive at statistics. Take the case of wool-spinning. The data that should be tabulated are—the weight and cost of raw wool used in a given time, in the aggregate, in each room, and by each mule; the weight of yarn produced, in similar detail, and the weight of waste material recovered; the price realized for the products (or, if the yarn is used in the same factory, the estimated value); and the cost in wages and in oil and sundries. Over a longer period an estimate should be made for the interest on the capital value and the depreciation of the machinery used, together with a proportional allowance for the general expenses of the factory, such as salaries, rent and rates, and advertising. The cost of the engine should be placed under the special expenses, if possible; if not, this cost must be divided between the various rooms with what accuracy is practicable. With such data it is possible to tell what

machines, rooms, or departments are running at a loss, or just paying their special expenses, or contributing adequately to the general expenses, or making a profit.

In this case, also, it is easy to state the number of lbs. of wool spun and the length of the yarn produced, and the actual work done by each group of operatives (the spinner and piecers at each pair of mules), which is, in fact, measured for the basis of piece-wages. It can be at once determined whether the machine (the spinning-mule) is being used efficiently.

Similarly in weaving, data are easily available for the product per loom, per operative, and per £ of wages paid, and the totals can be made for each weaving-shed and for the factory as a whole.

3. A more complicated problem is presented in railway working, and an example of the method of compiling statistics in use on many important railways in America, India and elsewhere is very instructive. The data are twofold, based respectively on the details of the train service, and on the quantity of goods conveyed; the first are connected with expense, the second with remunerative work done.

For each journey of each train the guard sends in a report as to the time at which the engine arrived, the times (actual and due) at which the train arrived at and left each stopping point, and as to the number of wagons (empty or loaded) hauled each section of the journey, with other details. For each journey of each engine the driver reports the time he was working with the engine, and how it was allotted to standing, shunting, or running, the amount of coal taken on, and the number of the wagons hauled in each section of the route.

On the other side, returns are made of all consignments of goods, showing the stations at which they were received and where they were delivered, and the money received for their transit.

From these data the following tables among others are compiled:—

STATISTICS OF OPERATION. GOODS AND MINERALS TOGETHER.
Months of June 1904 and 1905.

	No. of working-days.	Tons carried.		Ton-miles.	Average train-load.	Train-miles.	Average distance hauled.		
		Total.	Per working-day.				Goods.	Minerals.	Together.
		000's	000's	00000's	Tons.	000's	Miles	Miles	Miles
1904	26	4468	172	1026	100·3	1023	35·4	18·5	23·0
1905	26	4325	166	980	104·5	938	34·3	18·6	22·7

	Engine-hours.			Ton-miles per engine-hour.	Train-miles per train engine-hour.	Train-miles per single track-mile per working-day.	Ton-miles per route-mile per working-day.
	Train.	Shunting.	Total.				
	00's	00's	000's				
1904	1386	1922	331	310	7·38	13·2	2363
1905	1263	1821	308	318	7·43	11·8	2223

	Wagon-miles.				Average train-load of wagons.			Wagon-miles per engine-hour.			Average wagon-loads.
	Loaded.	Empty.	Total.	% Loaded of Total.	Loaded.	Empty.	Total.	Train per hour.	Shunting per hour.	Together per hour.	
	0000's	0000's	0000's								Tons.
1904	1939	1174	3113	62·3	18·9	11·5	30·4	224	162	94·1	5·29
1905	1821	1123	2944	61·9	19·4	12·0	31·4	233	162	95·4	5·38

"Ton-miles" form the principal measurement of the revenue-yielding work done by a railway so far as freight is concerned, and are obtained by multiplying the number of tons in each consignment by the number of miles it is carried. "Train-miles" signifies the aggregate of the miles run by trains; "engine-hours" the aggregate of the hours in which an engine was working with a train, distinguishing running from shunting. The total of wagon-miles is computed by multiplying the number of wagons moved by the number of miles run separately for every section at the beginning of which the composition of a train was altered,

and adding these products. These results, together with the total of the tons moved and the fixed information as to the track, are sufficient for the tables.

Let T be number of tons moved, T_m number of ton-miles, T_{rm} number of train-miles, E_t and E_s numbers of train and shunting engine-hours, W_l and W_e the number of loaded and empty wagon-miles. Then the average train-load is $\frac{T_m}{T_{rm}}$ tons; the average distance hauled is $\frac{T_m}{T}$; the average of ton-miles per engine-hour is $\frac{T_m}{E_t + E_s}$; of train-miles per train engine-hour is $\frac{T_{rm}}{E_t}$; the average train-load is $\frac{W_l + W}{T_{rm}}$ wagons; the average wagon-load is $\frac{T_m}{W_l}$, and the average number of wagon-miles per engine-hour is $\frac{W_l + W_e}{E_t + E_s}$.

Such figures could be worked out for any division of the railway that is required. By comparing the averages obtained for different months or different divisions, we can observe the work done by engines in hauling goods or wagons (ton-miles or wagon-miles per engine-hour), the use made of the track (or railway as ordinarily understood) and of double lines (train-miles per track-mile and ton-miles per route-mile), what proportion of haulage is effectively spent in hauling full wagons, and how heavily the wagons are loaded. Where any one of these averages increases, there is presumptive evidence of growing efficiency in working; where a difference or decrease is shown, there is a case for inquiry as to the cause; it may prove to be due either to the nature of the work, or to incompetency in handling it, or to a reorganization which produces a compensatory improvement elsewhere.

From similar tables the receipts per ton, per ton-mile and per train-mile are worked out for different classes of traffic.

4. In the case of railways and other large undertakings the problem is to discover exactly what measurement is most sensitive to efficiency of work, and to devise the necessary machinery for obtaining the statistics of precisely that measurement. In the running of goods trains the principal expense that can be reduced is the time during which the wages of the three men (driver, fireman and guard) concerned are paid; "wagon-miles per engine-hour" and "ton-miles per engine-hour" are found to provide precisely the tests wanted. In other cases it might prove to be the production per spindle per week, or the output of coal per hewer. When such tests are devised and kept systematically, an instant indication is given of any improvement or slackening in the work, and the reasons of the change can then be investigated.

5. It is clear that such a broad average as "wagon-loads" obtained by dividing 103 million ton-miles by 20 million loaded-wagon-miles does not satisfy the test of homogeneity suggested above (p. 65); a railway may be engaged in hauling coal by the train-load and also in handling small parcels for quick delivery; for the former heavy wagon-loads are easily obtained, with the latter the rapidity (and the custom) may be lost if goods are not forwarded till a wagon-load is ready. In other industries high average production may depend on inferiority of goods. Where the relative proportions of the different classes of work done vary very little, this consideration will not vitiate the comparison of averages; but where the proportions are not steady, further analysis and subdivision must be made, so far as practicable, till statistics are obtained for nearly homogeneous work; the first step made in this direction in railway statistics is in separating minerals from other goods. In the same way the analysis should extend, both for quantity and cost, to the smallest subdivision of the work that can be separated.

The labour and expense of collecting statistics in this way is much diminished if, when the actual averages or quantities which form the most delicate tests of efficiency have been decided on, no statistics are accumulated, which are not

directly needed for these averages, etc., and simple printed forms are used, which can be easily filled in an ordinary routine; these forms should be regularly delivered to a statistical clerk, who should systematically tabulate them on a uniform scheme.

6. There are two considerations which affect the use and formation of such statistics; first, the value of money is subject to continuous changes; secondly, it is not easy to find a common measure of the work done.

For the change in the value of money the reader is referred to Part II, Chapter IV, below, with the suggestion that special index-numbers should be formed to suit particular circumstances.

The addition and comparison of unlike quantities can often be made by the device of "weighted totals." This can be illustrated by the general statistics of the worsted trade.

EXPORT OF WORSTED TISSUES

	1894. Yards.	1907. Yards.	Mean price, 1894–1907. d. per yd.	"Weights."	Numbers adjusted to common measure. 1894.	1907.
	0000's	0000's			00000's	00000's
Broad coatings, all-wool .	1117	1379	46·2	20	2234	2758
„ „ mixed .	385	720	28 0	12	462	864
Narrow coatings, all-wool .	217	41	31·7	14	304	57
„ „ mixed .	272	159	20·3	9	245	143
Stuffs, all-wool . . .	1320	1043	11·9	5	660	521
„ mixed . . .	7756	6559	9·4	4	3102	2624
Total of Worsted Tissues	11067	9901			7007	6967
Ratio . . .	100 : 89·4			. .	100 : 99·4	
Total value .	£6,666,000	£7,394,000				
Ratio . . .	100 : 110 9					

During this period (1894–1907) the price of wool and of woven tissues fluctuated considerably, 1907 being a year of high prices. The aggregate value is therefore not a fair measure for comparison. The value in four of the six categories into which the exports are divided fell, and rose in the other two. The aggregate yardage fell. Now, a yard of

"Broad pure wool coatings" cannot properly be added to a yard of "mixed stuff"; the first is much heavier, broader and more expensive than the latter. The average prices of these six classes are shown in the table; the first is worth five times as much as the last per yard. Assume that these prices are proportional to the intrinsic values of (or to the work done in producing) the cloth, and for simplicity of computation take integers nearly in the proportion shown. These are called "weights" in the sense of Chapter III above, and it is known (pp. 18, 32) that they need not be taken with great accuracy for purposes of comparison. Multiply the quantities by the "weights," and so obtain the last two columns; here in effect the unit is "one quarter of a yard of mixed stuff" equal to $\frac{1}{20}$th of a yard of pure broad coatings, etc.

The comparison of the weighted totals shows that the total production was practically the same in 1894 and 1907 on this basis, though the value rose 11 % and the aggregate yardage fell 11 %.

The actual average weights of wool per yard in the various classes might be used for the statistical weights if they could be estimated. This method is used in the railway statistics of live stock, when one horse is counted as equivalent to so many sheep or to so many fowls, for purposes of transit cost, and it is capable of wide and varied application.

7. It is often useful to make and keep up to date charts of prices, cost, output, wages, etc., in considerable detail. In particular, if a trade is seasonal, it is well to have a graphic record of the seasonal fluctuations, with a view to forecasting the immediate future, and to providing an adequate supply for the probable demand.

It is generally interesting and sometimes of importance to preserve a record of the rates of wages paid to various classes of operatives, and also the average for the whole. It has frequently proved to be the case that the average has risen faster than the rates, owing to the different growths of various grades of labour and to readjustments of work. Such changes are often unobserved, but are frequently the main

factors in the growth (or, less frequently, the diminution) of earnings.

8. The principles of measuring the progress and efficiency of an institution are similar to those just outlined, but the statistical aspect is less important; for, while a commercial company is in business for the dollars and the test of success is pecuniary, an institution exists for carrying out some defined aim, for which there is in general no numerical measurement. Nevertheless it is more necessary to test the statistics offered by the management of an institution, especially when it is appealing for help, than those collected by a commercial body for itself; for it is in the interest of the latter to know the facts exactly, while the former needs to show a good case, and there is nothing so easy as to show a biassed result without actually falsifying the facts. In its own interest, for success in working, an institution should record its facts on a commercial basis, and in candour should present these records to the section of the public concerned. Hospitals, asylums, schools, colleges, and propagandist, religious, philanthropic and social societies are among the institutions to which these remarks apply.

As regards £ *s. d.*, accounts should be kept in great detail and carefully allotted to services and departments. In particular the expenses of advertising, of collecting money, of printing, of postage, and of administration, should be shown clearly, and separated from the expenditure directly on the objects for which the institution exists; the former correspond to the general expenses of manufacture. Further, when building, new or old, is involved, the exact state of the building account should be shown, and the amount spent on rent, interest, rates and taxes. When these things cannot be found clearly in a balance-sheet, suspicion always may arise that there is something to conceal. The proportions of the foregoing expenditures to total expenditure afford tests of the efficiency of administration that, when applied with knowledge of what has been done in similar cases, are very useful.

The costs of carrying out the objects of the institution should then be allotted, so far as they can be properly credited, to a department or group of departments. Averages should then be worked out—for a hospital, the cost of food and other household expenditure per head per week; for a school, the cost of teaching per child per term; and so on. At this point the question of homogeneity must be considered. The averages just mentioned would be useful in an asylum or workhouse or general hospital usually nearly full, and in a large primary school, but not in an institution where there were many grades of expense, or a college where there was specialized teaching for small classes; nor would one judge a missionary society by its expenditure per convert. The less an institution belongs to a regular type, and the less uniform the persons it deals with, the less, also, can general averages be usefully applied; but where it is possible to compare like with like, then the causes of differences in such averages should be sought out.

9. As regards statistics of results, of success in carrying out the declared aim, it is well to apply Rule 4 of the previous chapter; think out what is the exact measurement that is wanted. In a hospital the number of patients dealt with, together with the average length of stay,* and details of the number cured or relieved, should be known; the number of operations is often stated, but it may include the extraction of a tooth as equal to tracheotomy, and is not of much use. In an asylum the number of persons should be given classified by sex, age and length of sojourn. In a teaching institution the difficulties are greater. No sensible person regards examination tests as adequate. The number of registered students is misleading, as the amount of time nominally given and the regularity of attendance vary greatly; the information should rather be given in details showing (for example) the numbers of students, subdivided by age and standard of instruction, the number of classes

* A railway statistician would probably ask for "patient-days per bed."

per week attended, and a measure of the regularity of attendance; also the size of the classes should be stated. In some cases total teaching-hours, total student-hours, and student-hours per teacher may be stated with advantage; but these are likely to be misleading and suggest resemblance between railways and the business of teaching, which would only be found in a very wooden educational scheme.

A more useful way of studying such statistics is to compare them in detail year by year, and to try to account for the differences shown, remembering that the smaller the numbers dealt with the more apparent will be the variation from causes that are fortuitous and independent of the management of the institution.

In the end, statistics of this kind can only help to form judgments, which should be based mainly on non-statistical observation.

10. Our final subject in this chapter is the collection of data in connection with some social inquiry—for example, the amount of unemployment, the physique of children, the condition of a district as to overcrowding, or the more elaborate investigations that have been made as to general social conditions in London, York, Dundee, West Ham and Birmingham. The first thing to do is to think out in a quiet hour exactly what we desire to know, and, next, what part of this knowledge can rest on a statistical basis. For unemployment we might decide that the essential thing to discover was the number of hours' work obtained in the previous month, for overcrowding the number of cubic feet in a tenement per occupant, and so on, but we should at once find that additional measurements were necessary—*e. g.* in the last case the ages and sex of the occupants and the condition of ventilation. At this stage it is best to work out blank tabulations, where each column, row and total would give definite information on the subject of inquiry; then work out forms of questions, the answers to which would lead to the tabulation desired. Next consider what persons possess the information required.

The construction of the blank form of inquiry on which the answers are to be entered depends on the education and position of the people who are to fill it in. In general, it is useless to issue blank circulars unless the filling them in is compulsory. If the information already exists in written form, *e.g.* the record of wages paid at a factory, it can frequently be obtained by a personal visit at which the object of the inquiry is briefly explained, and interest aroused or at any rate consent obtained; and then a blank schedule carrying a clear explanation of what is wanted on its face, can be left. The questions must be such as can be answered by "yes" or "no" or in numbers; adjectives such as "fair," "occasional," etc., are nearly useless for tabulation, their significance varies from person to person. If, on the other hand, the data must be collected first hand, a house-to-house visit may be necessary. The labour may be abbreviated if the method of samples (Chapter VII above) can be strictly applied. Of course, tact and experience is necessary for this work. A separate blank form, again containing perfectly definite questions, should be used for each case; but except where measurements are necessary, the answers should be obtained in conversation and entered immediately afterwards; for a visitor taking notes is likely to be an object of suspicion.

11. The data having been collected, their working-up can be done in the light of the previous chapters. The special difficulty in this kind of investigation is the essential indefiniteness of the quantities (poverty, physique, etc.) to be measured. It is well not to draw a single definite line, and say above this line is health, below it weakness, or above this mark competence, below poverty, but to remember that health, poverty, unemployment, overcrowding, etc., are relative. The final statistical table should be a graduation—so many tenements where there was more than 500 cubic feet per person,* so many at 400 to 500 cubic feet, and so on. Then

* In this case a person should mean an adult, and children should be counted as fractions according to their age.

the effect of drawing the line at various grades can be observed.

All statistics which cannot bear full criticism should be put aside, even if the inquiry has to be given up; imperfect statistics on such questions are often only productive of harm. In publication, the whole method of inquiry should be clearly and frankly shown, the tabulations should be perfectly clear, and the statistics of the inquiry be definitely separated from other parts, which deal (for example) with supposed causes and suggested remedies. Space should not be wasted in printing elaborate tables of data, but enough detail must be shown to allow a critic to form an accurate judgment as to the adequacy of the inquiry.

PART II

CHAPTER I

THE POPULATION CENSUS

1. THE population of the United Kingdom has been counted once in ten years; the first Census was in 1801, the most recent complete Census is that of 1911. In 1921, the Census was taken in Great Britain, but not in Ireland; in 1923, Southern Ireland was separated from the United Kingdom, and, in 1926, separate Censuses were taken in North and South Ireland. Midnight before the first Monday in April had been the date taken in recent Censuses, but owing to the railway strike the date was postponed to June 19 in 1921. Blank forms are left with every householder, whose duty it is to enter certain particulars about every person dwelling in the house alive at midnight. Precautions are taken to avoid omissions and duplications, and persons not in houses are counted as far as possible. A supplementary test of population is afforded by the enumeration of the number of inhabited houses.

The population enumerated for a district is thus the number who happened to be there at a particular moment, which differs from the number who live there habitually and differs greatly in many important cases from the number who work there. The accidental element arising from absence on journeys or presence on visits is not important in most cases; but it is evident that the population of holiday resorts fluctuates greatly through the year, and that the selection of April is arbitrary, and it was found that the postponement to June in 1921 made a considerable difference in some cases.

The principal questions put to every person are as to age, sex, condition as to marriage (known as "civil condition"),

number of children, occupation, and birth-place. The number of rooms occupied by the family group is stated. The Royal Statistical Society has continually pressed for a more frequent census and for improvements in, and additions to, the questions asked.* Readers should compare the Census schedules of 1901, 1911, and 1921 with each other.

The organization of the Census and the working out and publication of the results are entrusted to the three Registrar-Generals of England and Wales, Scotland and North Ireland. The forms of questions and the methods of publication differ in the three countries. The principal general results for the United Kingdom are brought together in the General Report on the Census for England and Wales.

The Census of 1921 classifies the population according to its Administrative divisions. England and Wales are divided into 62 Administrative Counties,† and 82 County Boroughs, the latter being associated with the former in some totals and not in others. The Counties are divided into 1126 Urban Districts ‡ and 672 Rural Districts, which are again subdivided into Civil Parishes, unless one Civil Parish is coincident with the District. Some Urban Districts are distinguished as Municipal Boroughs. The A.C. of London is divided into the City of London and 28 Metropolitan Boroughs.§

For England and Wales the results of the Census of 1921 were published in the following method. A Preliminary Report showing the population for all Counties, Boroughs and Urban and Rural Districts (but not for Civil Parishes) was published nine weeks after Census day. There followed 4 volumes for London and 46 volumes each dealing with one, or occasionally two or three, Counties, published from October 1924 till July 1925, at which date General Tables for England and Wales appeared (price 13*s.*). Separate volumes dealing

* See *Statistical Journal*, 1908, pp. 496–8, 1909, pp. 574–93, and 1920, pp. 134–9.

† Abbreviations commonly used are A.C., C.B., U.D., M.B., R.D., C.P.

‡ Including C.B.'s, counting London A.C. as one.

§ See also "Note on Certain Divisions," p. 103 below.

with Occupations, Industries, Workplaces (with a supplementary part for London and five Home Counties), and Dependency were published in 1925–6, together with 3 index volumes, and an account of Ecclesiastical Areas. The series was completed by the General Report, which did not appear till late in 1927. Of this series the most important volumes are the General Tables and the reports on Occupations and Industries.

The publication of the Census of Scotland was more rapid, and was contained in 4 volumes, after a preliminary report.

The Census of the United States is also decennial, preceding that for the United Kingdom usually by nine months (June 1900, 1910, 1920). In using it, it is important to distinguish the Continental United States from the total, which includes outlying regions such as Alaska, Cuba and the Philippines.

2. The following table shows the growth of the populations of England and Wales, Scotland and Ireland, separately and together. As an example of further analysis the population of London and groups of manufacturing counties are also shown. In 1921, the population of the County of London was 4,483,000; its area is about 120 square miles, the great part of which, but not all, is thickly populated. Adjacent to it are populous Boroughs, such as Willesden, Tottenham, E. and W. Ham, Wimbledon, etc., and beyond these many other suburban areas. The table puts together the whole counties of Kent, Essex, Hertfordshire, Middlesex, Buckinghamshire, Berkshire and Surrey, for though a great part of these areas is rural, the growth in their population is mainly attributable to their neighbourhood to London. The group headed Northern consists of Cheshire, Lancashire, Yorkshire (West Riding), Durham and Northumberland; the Midland group contains the counties of Derby, Leicester, Nottingham, Northampton, Stafford, Warwick, Worcester, Monmouth and Glamorgan. It will be noticed that each of the selected groups increased about 150 per cent. between 1851 and 1921, while the rest of England and Wales together increased only 40 per cent.

GROWTH OF POPULATION
(0000's omitted)

	United Kingdom.	England and Wales.	Scotland.	Ireland.	London and Neighbouring Counties.	Mining and Manufacturing Counties. Northern.	Midland.	Rest of England and Wales.
1851	2737	1793	2889	6552	406	451	276	660
1861	2893	2007	3062	5799	469	529	321	688
1871	3148	2271	3360	5412	551	628	364	728
1881	3488	2597	3736	5175	651	757	427	762
1891	3773	2900	4026	4705	755	862	489	794
1901	4146	3253	4472	4458	874	976	568	855
1911	4522	3607	4761	4390	969	1084	655	899
1921	—	3789	4882	—	1014	1136	725	914
1926	—	—	—	— North 1256 South 2970	—	—	—	—

3. The areas of all districts, including the Civil Parishes, are stated in the County reports, and the density of the population (the number of persons per acre or other unit of area) can be worked out in fairly minute detail. It is important for this purpose to take sufficiently small areas, for it is evident that for most practical purposes the variation over a square mile is more important than that from county to county.

As an example of analysis by density we will assemble the statistics for "The City and County of Bristol" in 1901 and 1921. The ancient City of Bristol was situated in the County of Gloucester, which in this neighbourhood is separated from Somerset by the river Avon. In the City the original course of the Avon is difficult to trace, and long ago the City took in part of what had been Somersetshire. After many extensions of its boundaries to absorb Clifton (perhaps in the eighteenth century) and more recently the growing suburbs, its area was increased to 11,705 acres (15½ square miles) in 1901. Very soon after the Census of 1901, the County Borough of Bristol was extended to take in a strip of country on the right, or Gloucestershire bank of the Avon, about five miles along the river and two or three miles in breadth, so as to include the growing suburbs of Westbury-on-Trym and Horfield, and

beyond a long stretch of country the town and docks of Avonmouth, the property of the City, where the Avon reaches the Severn or Bristol Channel.

BRISTOL AND ENVIRONMENT

	1901			1921.		
	Acres.	Population.	Persons per acre.	Acres.	Population.	Persons per acre.
Bristol, City and County of; C.B. . . .	11,705	328,945	28·1	18,436	376,975	20·5
In Gloucestershire:						
Horfield U.D. . .	832	1,435	1·7	—	—	—
Barton Regis R.D.:						
Shirehampton C.P.	1,175	2,570	2·2	—	—	—
Westbury C.P. .	2,895	6,063	2·1	—	—	—
Henbury C.P. .	8,552	1,951	·2	6,999	2,260	·3
Warmley R.D.:						
Hanham Abbots C.P. . .	1,062	744	·7	1,062	1,040	1·0
Bitton C.P. . .	3,665	3,138	·9	3,665	3,235	·9
Mangotsfield C.P. .	2,564	8,806	3·4	2,564	10,720	4·2
Oldland C.P. .	973	1,905	2·0	973	1,933	2·0
Siston C.P. . .	1,833	1,352	·7	1,833	1,548	·8
Kingswood U.D. .	1,525	11,961	7·8	1,530	12,951	8·5
In Somersetshire:						
Keynsham R.D.:						
Brislington C.P. .	1,783	2,091	1·2	1,783	3,493	1·9
Keynsham C.P. .	4,235	3,152	·7	4,235	3,837	·9
Long Ashton R.D.:						
Long Ashton C.P.	4,193	2,023	·5	4,193	2,218	·5
Abbots Leigh C.P.	2,276	327	·1	2,276	595	·3
Easton-in-Gordano C.P. . .	1,820	2,284	1·3	1,765	2,509	1·4
Portbury C.P. .	2,847	398	·1	2,817	483	·2
Portishead U.D. .	1,036	2,544	2·4	1,029	3,815	3·7
	000's	000's	000's	000's	000's	000's
Gloucestershire, A.C., excluding Bristol .	794	379	48	787	381	·48
England and Wales .	37,327	32,528	87	37,340	37,885	1 01

In the table above the Rural Districts which surround Bristol are also shown, so as to include all populous places within seven miles of the centre. In Gloucestershire, Mangotsfield is a large village by a railway junction with easy access to Bristol, and Kingswood is an old and unprogressive town

principally devoted to Boot Manufacture. In Somersetshire, Brislington is adjacent to South Bristol, and the Long Ashton R.D. stretches along the left bank of the Avon as far as the Bristol Channel. Abbots Leigh is contiguous with that part of Clifton which has spread across the river. Easton contains the little town of Pill, which is connected by ferry with Shirehampton and Avonmouth. Portishead is a minor semi-seaside resort on the Bristol Channel.

In purely rural districts in the South of England the density is usually about ·25 per acre (160 per square mile). Where the density is over 1 per acre the district generally becomes Urban, and between these limits there are generally urban or suburban characteristics or some special local industry or some small nucleus of population. It is evident that there are few suburbs of any importance that had not by 1921 been absorbed by the County Borough.

Within Bristol itself, however, there are very striking differences in density. Apart from the Ward of Horfield (a suburb on the Gloucester Road), and the Ward of Westbury (which includes Avonmouth), added in 1901–2, the density varies from 17·3 in Central West, which includes the old Docks, Warehouses, etc., and 18·0 in S. George East, to 107 in S. Paul's Ward.* It falls, in fact, rapidly from the region which comprises old Bristol and the first extensions eastward and across the river to Bedminster, to the outskirts. In Clifton itself, where the Avon flows through a gorge towards Avonmouth, the density is 20 in the North, which contains the old residential or health-resort, but 39 in the South, where the houses are crowded in the lower ground by the river.

Now an uninformed inspection of the statistics would result in the statement that the density of the population of Bristol had diminished from 28 persons per acre in 1901 to 20·5 in 1921. Actually the density on the area which was Bristol in 1901 has increased from 28 in 1901 to nearly 31 in 1921, while that of the added area of Westbury, etc., has also increased.

* Southwark and Shoreditch, London, had densities 163 and 159 respectively in 1921.

BRISTOL IN 1921

	Acres.	Population.	Density.
Central Wards:			
Clifton South	239	9,300	38·9
S. Michael	267	11,669	43·7
District	342	17,717	51·8
S. Paul	160	17,133	107·0
S. James	133	10,075	75·8
S. Augustine	320	16,721	52·3
Central, E.	109	3,779	34·7
Central, W.	72	1,248	17·3
Redcliffe	245	7,606	31·0
S. Philip, N.	246	22,053	89·6
S. Philip, S.	269	19,576	72·8
Wards S. of R. Avon:			
Bedminster, E.	615	20,639	33·5
Bedminster, W.	1,131	23,656	20·9
Southville	250	19,697	78·8
Somerset	1,108	21,677	19·6
Eastern Wards:			
Easton	252	22,679	90·0
S. George, W.	483	22,906	47·4
S. George, E.	1,348	24,305	18·0
Stapleton	2,573	28,051	10·9
Northern Wards			
Horfield	1,314	19,571	14·9
Clifton North	443	8,930	20·2
Redland	501	12,217	24·4
Westbury	6,016	15,770	2·6
Total	18,436	376,975	20·5
Total, excluding Horfield and Westbury	11,106	341,734	30·8

The distinction between the area comprised in a Borough or other Urban District and that in a Rural District is mainly one of administrative convenience. The towns constantly grow past old boundaries, and neighbouring villages assume urban characteristics while still in the midst of agricultural country; till, suddenly, the town extends and includes not only the now populous villages but a great stretch of country as well.

4. The distinction between Urban and Rural population is arbitrary in any one country, but the uncertainty is greatly increased when we compare the apparently similar classification in two countries. The general difference in method can

be illustrated by studying the information available about one of the principal American cities, which in history and situation has some points of resemblance to Bristol.

Boston, Massachusetts, is at the head of a deep bay, partly enclosed by Cape Cod, and the older portion is principally on the north side of the short estuary of a river which forms a natural harbour. The City Proper has extended to include an area of 43½ square miles, and adjacent to it a considerable number of " cities " have developed, of which the best known is Cambridge, in which is situated the University of Harvard. These and a considerable part of the Counties in which they stand are included in the Metropolitan District of Boston, which, as a whole, contains 570 square miles. The Census (U.S. Census for 1920, Vol. I., p. 63) places with this area a further stretch of adjacent territory containing about 42 square miles. The only data relating to area are contained in the following table.

BOSTON, MASS., AND ENVIRONMENT.

	Area (acres).	1910. Population.	1910. Density. Persons per acre.	1920. Population.	1920. Density. Persons per acre.
Metropolitan District:					
City Proper .	27,870	670,585	24·1	748,060	26·8
Outside City .	337,203	860,553	2·6	1,024,194	3·0
Adjacent Territory:					
Outside Metropolitan District .	26,943	25,533	·9	29,066	1·1
Total . .	392,016	1,556,671	4·0	1,801,320	3·6

Population in principal places in the Metropolitan District outside City 1920 :—

	000's		000's
Middlesex County:		Essex County:	
Cambridge City . .	110	Lynn City . . .	99
Somerville City . .	93	Others . . .	90
Others . . .	363	Suffolk County . .	87
Norfolk County . .	182		

Middlesex is East of Boston, Norfolk is South, Essex is North, and Suffolk is the County out of which the newer parts of Boston have been cut.

5. Closely connected with the distribution by locality is the distribution by occupation. This classification is extremely difficult, and it is prudent to take only those comparisons which are given in the Census volumes, and to regard even them with suspicion, unless one has time to go into the question in minute detail, reading the text of the General Report for each Census, and studying the changes in classification.

In using the table on p. 94 it must be realized that the figures are per 1,000 of the selected part of the population, not absolute numbers, and that one division can only grow at the expense of another. For example, the actual number of females working in connection with Textile Fabrics was greater in 1901 than in 1881. Further, it must be remembered that the groups are not homogeneous (see p. 65 above) either in age or in occupation. The number of occupied children tends to diminish as educational requirements are enforced; this accounts, for example, for part of the diminution under the heading "agriculture." The table is greatly contracted, and only suggests broad outlines for investigation.

Under the heading "Professional, etc." are included those engaged in government, central or local, and their subordinates, the army and navy on land or in port, and members of the professions and their assistants. "Domestic" excludes gardeners and coachmen, but includes a growing number of laundry-workers, lift-attendants, etc. "Commercial" includes merchants, dealers, "travellers," and clerks. "Transport" includes railways (but not railway construction), roads, rivers, docks and the telegraph and telephone services. [By the grotesqueness of the Census tabulation the Post Office comes under heading I, 1. "National Government."] "Metals" includes all work in metals, except mining, and the manufacture of tools, machinery and engines, ships and carriages. "Building" includes navvies and road labourers. Sailors and soldiers are only included in the Census enumeration when on land or in port at or within a few days of the

date of the Census. "Undefined" includes a diminishing number of agricultural and builders' labourers.

The residual heading in the Census, "Without specified occupations or unoccupied," has no relation whatever to "unemployed"; it included, in 1911, among the males in England and Wales 352,000 persons retired from business,* 84,500 pensioners, including old age pensioners, 52,000 "living on their own means," and 1,700,000 others, "including students." The number of women "without specified occupations" is of course very much greater.

GROUPS OF OCCUPATIONS IN THE UNITED KINGDOM

	Per 1,000 MALES over 10 years.			Per 1,000 FEMALES over 10 years.		
	1881.	1901.	1911.	1881.	1901.	1911.
Professional, etc.	46	52	59	17	23	26
Domestic	8	9	11	142	122	110
Commercial	30	41	45	1	5	9
Transport	75	95	97	1	1	2
Agriculture	188	136	125	16	9	6
Mining	49	60	70	—	—	—
Metals	75	91	97	3	4	5
Building	74	86	70	—	—	—
Textiles	48	38	39	61	52	51
Dress	35	32	30	59	54	48
Food and Lodging	54	60	62	15	22	29
Other Manufactures, etc.	61	74	77	11	16	18
Undefined	85	61	50	8	8	10
Total occupied	827	834	832	335	316	315
Retired or unoccupied	173	166	168	665	684	685
	1,000	1,000	1,000	1,000	1,000	1,000
Actual total number of persons over 10 years (0000's omitted)	1255	1554	1719	1350	1680	1856

6. In 1901 and earlier Censuses the classification was partly with reference to the particular craft or occupation a person followed, partly with reference to the industry in which he was engaged, so that as a result there was no purely occu-

* Other than the Army, Navy, Church or Medicine; these are tabulated under their professions.

pational or purely industrial analysis. In 1911, the former classification was followed, but there was also an industrial classification made. The distinction and method can be explained by the following example.

ENGLAND AND WALES, COTTON INDUSTRY, 1911. MALES.
(Census, Vol. X, Part I, pp. 582–3)

Classified under Cotton Manufacture in Occupation Tables		233,380
Additional workers in the industry:		
Classified under Engineering, etc.	4,007	
„ „ Building, etc.	570	
„ „ Engine drivers, etc.	4,532	
„ „ Clerks, etc.	5,796	
„ „ Transport	2,312	
Others	554	
		17,771
		251,151
Less persons in other industries classified under Cotton occupations		160
Number in Cotton Industry		250,991

Here the additional workers are employed directly by firms manufacturing cotton, but were classed as clerks, carpenters, stationary engine-drivers, etc. in the former tables.

In 1921, the whole classification was revised, with the effect of making comparisons on the earlier basis generally impossible, but allowing broad comparison with the Industrial tables of 1911. The occupational classification was separated completely from the industrial, and the results published in different volumes; but in the Industrial volume a considerable amount of detail is given of the numbers of persons in particular occupations included in each industry. It is possible to construct a table for industries similar to that given for occupations above for England and Wales, but not for the United Kingdom, for 1911 and 1921. The differences between the columns for 1911 in the two tables are principally due to the omission of Ireland, which is primarily agricultural, in the second.

ENGLAND AND WALES.

Industrial Groups.	Per 1000 Males over 10 years. 1911.	1921.	Per 1000 Females over 10 years. 1911.	1921.
Professional, etc.	74	91	31	40
Domestic	28	23	107	71
Commercial	121	105	51	45
Transport	81	80	1	2
Agriculture	83	71	7	5
Mining	82	87	—	—
Metals	107	134	8	15
Building	63	51	—	—
Textiles	38	33	44	40
Dress	25	21	47	31
Food and Lodging	39	35	27	31
Other Manufactures, etc.	70	74	17	23
Undefined	27	23	5	5
Total occupied	838	828	325	308
Retired or unoccupied	162	172	675	692
Total	1,000	1,000	1,000	1,000
	0000's omitted.		0000's omitted.	
Actual numbers over 10 years old	1366	1463	1486	1642

It is possible to construct the table on p. 97 on similar lines from the Statistical Abstract of the United States (1926, pp. 48–9). Though the definitions and classification differ, some broad generalisations could be made in a comparison of the distribution among industries in the two countries.

7. Apart from the Census we find in the XVIIIth Abstract of Labour Statistics (pp. 31 *seq.*) details of the numbers employed on Ships, in Agriculture, at Mines and Quarries and by Railway Companies at various dates. More general statements are given currently in the *Ministry of Labour Gazette* of the numbers insured in industries, tabulated nearly in accordance with the Census of Population tabulation, but not available for the Census date, and in the Reports of the Census of Production, 1924 (see Chapter V below). The Population Census includes all occupied, whether employers or employed; the Census of Production includes all employed; the Insurance

CONTINENTAL UNITED STATES

Class of Occupation.	Per 1,000 Males over 10 years. 1910	1920.	Per 1000 Females over 10 years 1910.	1920.
Professional and Public service	38	44	22	26
Domestic and Personal	34	29	74	54
Clerical	31	40	17	35
Trade	85	85	13	17
Transportation	68	68	3	5
Agriculture	293	233	52	27
Extraction of Minerals	26	26	0	0
Manufacture	238	257	53	48
Total occupied	813	782	234	211
Unoccupied	187	218	766	789
Total	1,000	1,000	1,000	1,000
	000's omitted.		000's omitted.	
Actual numbers over 10 years	3703	4229	3455	4045

numbers include all manual workers over 16 years old, and other employees receiving less than £250 per annum, and is extended to North Ireland,* while the other accounts relate to Great Britain only. Only a very rough agreement between these accounts is to be expected, but it is worth while to bring them together, since they arise from completely different sources—viz. householders' statements, employers' statements, and the Labour Exchanges.

NUMBERS OCCUPIED IN SELECTED INDUSTRIES

(000's)

	Population Census.	Insured Persons.	Census of Production.
Coal and Shale Mining	1,305	1,260†	1,176
Cotton	621	562	517
Wool and Worsted	260	261	276
Silk	34	42	39
Dyeing and Finishing	117	118	108

* In the table, North Ireland is, in fact, represented by only about 6,000 persons under Dyeing and Finishing.

† Including about 75,000 unemployed.

8. The Census affords a test of the amount of crowding in houses by the rough classification of the numbers of persons per room.* The term "overcrowded" used to be employed technically to designate the condition of more than two persons per room, but since more scientific tests are devised (*e.g.* number of cubic feet per head in sleeping-rooms), it is well to avoid the word. The following table summarizes part of the information available. Since in tenements of six rooms and over there is more elasticity of accommodation, the relatively small number of these in which there are more than two persons per room is omitted, except in the last line. The numbers exclude the population enumerated in institutions, etc., and include all denominated "private families" in the Census.

DENSITY OF OCCUPATION OF TENEMENTS, ENGLAND AND WALES, 1921

Number of Tenements.

Tenements of	Occupied by	County of London.	County Boroughs.	Other Urban Districts.	Rural Districts.
		00's.	00's.	00's.	00's.
1 room.	1 or 2 persons	1,182	664	410	86
	More than 2	296	317	177	43
2 rooms.	4 persons or fewer	1,949	2,523	2,186	947
	More than 4	419	571	413	171
3 rooms.	6 persons or fewer	2,362	4,071	3,467	2,186
	More than 6	266	598	416	221
4 rooms.	8 persons or fewer	1,914	6,984	7,028	4,696
	More than 8	106	290	273	151
5 rooms.	10 persons or fewer	997	6,072	6,838	4,056
	More than 10	15	67	71	34
(*a*) Total number of tenements of all sizes		11,209	28,685	29,452	18,046
(*b*) Number of "overcrowded" tenements		1,102	1,843	1,350	620
(*b*) as per cent. of (*a*)		9·8	6·4	4·6	3·5
Per cent. of population, more than 2 to a room, in all tenements		16·1	10·9	8·1	6·5

* Or, as in the Census volumes of 1921, of rooms per person.

9. The population in 1911 is of course equal to that of 1901,* together with the number of births and immigrants between the Census dates, less the number of deaths and emigrants. The emigration statistics for the United Kingdom as a whole were not till 1908 adequate for such estimates. We have rather to work backwards to find the net result of migration and travelling.

United Kingdom.—	000's
Population, 1901	41,459
Births, 1901–11	11,614
	53,073
Deaths, 1901–11	6,771
Population in 1911 if no migration . . .	46,302
Enumerated population	45,222
Deduced excess of emigrants over immigrants .	1,080

The excess of the number of births over deaths is called the " natural increase of population."

It is necessary for many purposes to estimate the population at intermediate dates. The most accurate method is to make the best estimate possible from the migration statistics, whose effect can be checked every ten years, and combine these with the recorded numbers of births and deaths.

Another way is to assume that the population increases continually in geometric progression; this rate was equal to ·89% per annum for the United Kingdom between 1901 and 1911, and to ·95% between 1891 and 1901; it is clear that some process of " smoothing " is necessary to pass from one rate to the other in 1901.

The following table shows various methods of estimating the numbers for the United Kingdom at the middle of each year, 1901 to 1911.

* The decade 1911 to 1921 is not taken because of the difficulty of measuring the movements and deaths due to the War.

Population in 1901 .	41,458,721	Logarithms .	7·6176160
1911 .	45,221,615		7·6552501
Excess . .	10) 3,762,894		10) ·0376341
	376,289		·0037634

April.	Arithmetic progression. 000's.	Geometric progression.		Computed from Births and Deaths less 108,000 net emigration annually. 000's.	Official estimate adjusted to April 1st. 000's.
		Logarithms.	Numbers. 000's.		
1902	41,835	7·62138	41,820	41,811	41,804
1903	42,211	7·62514	42,183	42,194	42,158
1904	42,588	7·62891	42,551	42,591	42,520
1905	42,964	7·63267	42,921	42,963	42,888
1906	43,340	7·63643	43,295	43,347	43,265
1907	43,716	7·64020	43,671	43,724	43,643
1908	44,093	7·64396	44,051	44,093	44,026
1909	44,469	7·64772	44,435	44,478	44,419
1910	44,845	7·65149	44,822	44,852	44,816

The first column assumes equal annual increments of 376,000 persons; the second method assumes an annual rate ·89% (log 1·0089 = ·00376); for the third method the births and deaths and average migration are as in the table just above. It is not stated how the official estimate is obtained.

The first method is the most rapid, and agrees with the others within 2 per 1,000. The more involved method, combining numbers of births and deaths and emigrants, is likely to be the most correct, if the migration figures are studied more minutely.

Any of the above methods can, and one or other must, be used for estimating the inhabitants of a county district or town; the "natural" increase is known from registration, but here is grave risk of error due to migration. The difficulties are accentuated when we estimate the population in (say) 1920, before we have the Census of 1921. We may, however, take the "natural" increase, and compare it with the increase that the previous intercensal rate of growth shows; we can base another estimate on the number of school children; and in some cases check the result from the number of houses

rated, but this is difficult. If these four methods agree, our estimate is good; their disagreement is a measure of the inaccuracy of the result. Local knowledge will sometimes allow the better of the four estimates to be chosen.*

10. The previous paragraphs deal with only a few of the very large number of problems and results of interest that arise from the census volumes. In conclusion, we will deal very briefly with the statistics of age. Age is stated inaccurately for the very young (through misreading of the instructions), for the very old (through ignorance or through the desire to magnify old age), and by women who are unwilling to confess even under the cover of secrecy to advancing age, and generally there is a tendency to return the age at the nearest round number, instead of at the last birthday; to correct this the age was asked in years and months in 1921. There may be a tendency to overstate age, with the idea that an old age pension may depend on it.

To overcome the concentration at round numbers, ages are tabulated as between 25–35, 35–45, etc. The other mistakes cannot be completely rectified, but they can be checked by two different methods. First, there is the record of persons at the various ages at all the previous Censuses, and the registers of deaths according to age and of births; from these the number surviving can be estimated, and the differences found must be attributed to migration † or mis-statement of age; the diagrams in the General Report of the Census, pp. 64 *seq.*, show the existence of some of the mis-statements already named, but in general confirm the accuracy of the answers. Secondly, it is certain that in a large population the numbers at successive ages must result in a nearly continuous and regular group; there cannot be a great number at 30, and relatively few at 29 and 31, unless there were great variations in the numbers of births about 30 years earlier. The applica-

* Students who wish to study the methods in use should consult papers by Mr. Waters, p. 293, and by Mr. Hayward, p. 434 of the *Statistical Journal*, 1901, and follow up the references there given.

† Or temporary absence in the case of soldiers.

tion of this principle is the basis of the life table, the survival table, the tabulated death-rates according to ages, and the other tables which supply actuaries with material for their calculations. The method of smoothing in the diagram, p. 37, depends on the same idea. It is beyond our scope to discuss here the mathematical methods which are employed. The Registrar General's Decennial Supplement, 1921, gives the result of the "graduation" for the whole population year by year from 0 to 100 years.

The following contracted table is important as showing the relative number of young and old, and of the two sexes, and the considerable modification between 1901 and 1921.

AGE DISTRIBUTION IN ENGLAND AND WALES, 1901 AND 1921

Numbers per 1,000 of all enumerated.

Ages.	1901.		1921.	
	Males.	Females.	Males.	Females.
Under 5 years . . .	57·0	57·2	44·4	43·3
5 and under 15 . . .	105	105	95	94
15 „ 25 . . .	95	101	84	92
25 35 . . .	76	85	69	83
35 45 . . .	59	64	66	75
45 55 . . .	43	46	56	60
55 65 . . .	27·9	31·8	36·5	40·4
65 „ 75 . . .	14·7	18·4	19·3	24·1
75 and over . . .	5·7	7·9	6·7	10·6
	483·5	516·5	477	523

These figures may be compared with a similar tabulation for the United States (*Statistical Abstract for U.S.*, 1926, p. 5).

AGE DISTRIBUTION IN THE CONTINENTAL UNITED STATES, 1900 AND 1920, AND IN THE UNITED KINGDOM

	United States, 1920.					United Kingdom, 1921.	U.S., 1900.	U.K., 1901.
			All					
Ages.	White.	Negroes.	Males.	Female	Total.			
0– 5 . .	109	109	56	54	110	*88*	122	*114*
5–15 . .	206	240	105	103	208	*189*	224	*210*
15–45 . .	472	487	239	234	473	*469*	476	*480*
45 and over .	213	164	110	99	209	*254*	178	*196*
	1,000	1,000	511	490	1,000	*1,000*	1,000	1,000

NOTE ON CERTAIN DIVISIONS according to which the population is, or has been, tabulated in the Censuses of England and Wales.

Ancient Counties are the old counties, 40 in England, 12 in Wales, which have been only slightly changed in historical times by the merging of their detached parts in the counties by which these are surrounded; in the case of Worcestershire considerable parts are still detached.

London was constituted as a separate administrative county, carved out of Middlesex, Kent and Surrey, in 1888.

Registration Counties are groups of registration districts, covering to a great extent the same areas as the Ancient Counties by whose names they are called. The *registration districts* are simply the Poor Law Parishes and Unions utilized for registration purposes, births, marriages and deaths, as well as census enumeration and tabulation. In connection with the Poor Law Reforms of 1834 parishes were grouped into Unions for Poor Law purposes round convenient centres, and county boundaries were generally ignored. Consequently the groups of registration districts which form a registration county overlap the ancient county boundaries seriously; for example, the populations of the Ancient and the Registration County of Derbyshire were 620,000 and 490,000 respectively in 1901,* but in most cases the differences are less considerable.

The registration districts are divided into *sub-districts*, and each sub-district is made up of one or more civil parishes. The *civil parish* is the smallest unit for Poor Law administrative purposes, but is not used for registration.

The statistics relating to the registration counties used to be summarized for some purposes in eleven Divisions, viz. London, South-Eastern, South Midland, Eastern, South-Western, West Midland, North Midland, North-Western, Yorkshire, Northern, and Welsh. In 1921 they are given a subordinate place (*e.g.* General Tables Volume, pp. 51 *seq.*, under Poor Law Union Counties).

* Summary Tables of the Census, Table II, 1901.

Administrative Counties. These date from the Local Government Act of 1888, which established County Councils. Several of the old counties were divided for this purpose into two or more administrative counties (*e.g.* The Parts of Holland, of Kesteven and of Lindsey in Lincolnshire, East and West Sussex), so that there are now 50 altogether in England and, as before, 12 in Wales. Boroughs which contained over 50,000 persons in 1881, and a few others which had before enjoyed some independence, were left outside the administrative counties and called *County Boroughs;* other boroughs which have since 1881 successfully claimed the possession of a population of 50,000 have been raised to the same rank. There were 75 county boroughs in England and Wales in 1911, and 82 in 1921. Many minor adjustments of county boundaries were made, but, except for the separation of London, the administrative counties (when the subdivision, as in Sussex, is ignored), together with the county boroughs they surround, are nearly co-extensive with the Ancient Counties.

Each administrative county (except London) is divided into *Urban* and *Rural Districts.* The urban districts are either boroughs or simply urban districts.* *Boroughs* are cities or towns which have been incorporated; each has a city or town council consisting of the Mayor, the Aldermen and the Councillors, whereas each other urban district has an urban district council with chairman and councillors. Most independent towns of considerable size or of ancient origin are incorporated. In the Census Reports boroughs are distinguished as C.B. (county borough) or M.B. (municipal borough), but strictly the latter include the former. Other urban districts are those regions which have been constituted as such, because of their density of population or of their urban

* The county boroughs are sometimes classified with, sometimes apart from, urban districts. Also they are sometimes included in and sometimes excluded from administrative counties in summary statistics. The County Borough of York stands partly in each of the three Ridings. Great care is necessary in reading the headings of tables on these accounts.

character, from time to time by the Local Government Board; they have special powers of administration, chiefly for sanitary and engineering purposes; the most populous of them are on the growing outskirts of boroughs in which it is their destiny to be included, others are mining or scattered manufacturing districts.

The boroughs and other urban districts having been subtracted from the county, the remainder consists of *rural districts*, each of which possesses a rural district council. Each urban and each rural district consists of a *civil parish* or group of civil parishes; the parishes in the rural districts have some powers of self-government exercised through the parish councils.

Civil parishes are thus grouped together in one way to make urban and rural districts and in another to make registration sub-districts. An urban district is in general part of a registration sub-district; a rural district is in general the remainder of a registration district when the urban districts, if any, are subtracted, the main exceptions being when the registration district is divided by the boundary of an administrative county.

London, for which the administrative and registration counties coincide, is under special laws; it consists of the City of London (with its Lord Mayor) and the City of Westminster and 28 *Metropolitan Boroughs* (each with a Mayor).

For most practical purposes the administrative counties and county boroughs have superseded the Ancient Counties. Birthplaces, however, used to be recorded for the census according to the latter.

The boundaries of civil parishes have been adjusted for this grouping into districts. *Ecclesiastical parishes* may either coincide with ancient or with new civil parishes, or they have been formed by subdividing former parishes, or by carving out a new parish when the population required it.

The division into parliamentary *constituencies* does not necessarily coincide with any of the divisions already named.

CHAPTER II

VITAL STATISTICS *

1. THE most easily accessible source of complete statistics of births, marriages and deaths is the Registrar-General's Annual Report, now *Statistical Review*.† The extracts from it in the Statistical Abstract are insufficient for many purposes. The sources of the Registrar-General's statistics are the familiar marriage and death certificates and register of births, filled in by those responsible on these important occasions. The registration districts have been the same as those used in the population census. [See Note, p. 118.]

Birth- and death-rates are obtained by multiplying the number of births and deaths recorded in a year in a district, great or small, by 1,000 and dividing by the estimated population of the district; the resulting rates are generally given to one place of decimals (thus: 15·3 per 1,000), and in the last chapter it was seen that the population of the whole of England and Wales, at any rate, could be estimated with sufficient accuracy. The marriage-rate is obtained by multiplying the number of marriages by two to get the number of persons and proceeding as before.

* Readers who desire more than this very slight summary should consult *Vital Statistics*, by Dr. Newsholme, Medical Officer of the Local Government Board. See also Dr. Newsholme's and Dr. Dudfield's papers in the *Statistical Journal*, 1905, 1906, and 1908, and Bertillon's *Cours elémentaire de Statisque Administrative*, Chs. VII, XIII, and XXVI-XXXII.

† There are also weekly and quarterly reports and an annual summary for London and large towns; and a Decennial Supplement (of which the last was published in 1928), giving comparative statistics and much detailed information in Part I, and the relation of deaths to occupations in Part II. The reports of local Medical Officers of Health for districts throughout the country may be consulted with advantage.

2. The following table shows how these birth- and death-rates have fallen in recent years in England and Wales. Similar phenomena are observed in most civilized countries.

ENGLAND AND WALES

Rates per 1,000 of the Population.

	Births.	Deaths.	Marriages.
1871–75 Annual average .	35·5	22·0	17·1
1876–80 „ „ . .	35·4	20·8	15·3
1881–85 „ „ . .	33·5	19·4	15·1
1886–90 „ „ . .	31·4	18·9	14·7
1891–95 „ „ . .	30·5	18·7	15·2
1896–00 „ „ . .	29·3	17·7	16·1
1901–05 „ „ . .	28·2	16·1	15·6
1906–10 „ „ . .	26·3	14·7	15·3
1911–13 „ „ . .	24·0	13·8	15·5
1922 „ „ . .	20·4	12·8	15·7
1923 „ „ . .	19·7	11·6	15·2
1924 „ „ . .	18·8	12·2	15·3
1925 „ „ . .	18·3	12·2	15·2
1926 „ „ . .	17·8	11·6	14·3
1927 „ „ . .	16·7	12·3	15·7

It is believed that births are adequately registered, but the possibility should be borne in mind that the regulations put in force in recent years for the immediate notification of a birth may bring the registration more up to date.

3. In the United States the registration of births and of deaths is incomplete, and was organized in 1925 in only 34 out of the 49 Continental States for births, and in 42 for deaths.

NON-REGISTRATION STATES

Neither Births nor Deaths registered.	Deaths only registered.
South Dakota.	Missouri.
Arkansas.	Tennessee.
Oklahoma.	Alabama.
Texas.	Idaho.
New Mexico.	Colorado.
Arizona.	Louisiana.
Nevada.	Georgia.
	S. Carolina.

Most of the non-registration States are in the central region, but there is no uniformity in their geographical distribution.

PERCENTAGE OF POPULATION IN REGISTRATION STATES

	Births.	Deaths.
1920	60	82
1925	76	89

Only rough and uncertain generalizations are possible from these data to estimates of the United States as a whole, but it is interesting to reverse the process of p. 99, and to compute the natural increase in the United States from the Census and migration statistics.

CONTINENTAL UNITED STATES

	000's.	000's.
Population, Census 1910		91,792
Gain by arrivals, 1910–20:		
American citizens	2,011	
Aliens	7,113	
		9,124
		100,916
Loss by departures:		
American citizens	2,461	
Aliens	3,988	
		6,449
		94,467
Population, Census 1920		105,711
Hence natural increase		11,244

The population midway between the Censuses may be put at about 98,750,000, and the rate of natural increase, therefore, at 114 per 1,000 for ten years, and 11·4 per 1,000 per annum.

Now the mean death-rate in the registration-area in 1910–20 was 14·3, and if this could be applied to the whole country we should have the birth-rate = rate of natural increase plus death-rate = 11·4 + 14·3 = 25·7. In fact, the birth-rate in the registration area was 24·7, 24·6, 22·3, 23·7 in 1917, '18, '19, '20, average 23·8. The figures are, therefore, not inconsistent with each other, if the birth-rate was higher before 1917 than after.

[The data are computed from the *Statistical Abstract of the United States*, 1926, pp. 3, 73, 80, 87 and 98.]

4. The fall of the birth- and death-rates since 1870 in

England and Wales has resulted in the very marked changes in distribution by age shown in the table on p. 102 above, and this change in distribution has, in its turn, affected the death-rate, since for different ages the chance of death varies greatly.

DEATH-RATES AT VARIOUS AGES

Per 1,000 living in each age group.

	England and Wales.				United States, Registration Area.	
	Average, 1901-1905.		Average, 1924-1926.		1920.	
Age Group.	Male.	Female.	Male.	Female.	Male.	Female.
0– 5 . .	53·8	44·9	24·6	19·9	21·8	17·5
5–10 . .	3·6	3·7	2·5	2·3	2·3	1·9
10–15 . .	2·1	2·2	1·6	1·6	2·0	1·6
15–20 . .	3·2	3·0	2·8	2·5	3·4	3·2
20–25 . .	4·3	3·6	3·3	3·0	4·3	4·3
25–35 . .	5·9	5·0	3·8	3·4	4·9	4·8
35–45 . .	9·9	8·2	6·3	4·8	7·8	6·9
45–55 . .	17·3	13·3	11·4	8·5	13·2	11·4
55–65 . .	32·9	25·7	24·2	18·1	27·0	23·2
65–75 . .	67·2	56·4	58·8	45·8	61·5	53·5
75–85 . .	137·4	121·5	136·8	113·3	145·7	137·2
85 and over .	283·0	261·3	281·1	257·8		
All: Crude .	17·1	15·0	12·7	11·3	12·2	10·5
Standardized * .	17·4	14·6	11·6	9·5	11·8	11·1

The death-rate is greatest in the first few months of life, falls rapidly to a minimum in childhood, and increases gradually till old age approaches, when it rapidly becomes great. It is at nearly all ages less for females than for males, so that though there are 4 or 5% more male births than female, the actual number of females is greater than that of males at all ages after about 10 years in a normal population. The general death-rate of a population is greatly affected by the relative number of the very young and of the old to the total, and of the relative numbers of the two sexes. Hence to make valid comparisons between the death-rates of two populations it is

* Standardized at the age and sex-distribution of England and Wales, 1901.

necessary to eliminate the variation of age and sex. This process is accomplished by choosing a particular distribution of age and sex as a standard, and then computing what would have been the general death-rate, for example, in the United States in 1920, if the death-rates in each age group were as recorded, but the age and sex distribution the same as in the standard population. In the table above, these rates are computed, when the population of England and Wales in 1901 is taken as the standard. The standardized rates for England and Wales are stated by the Registrar-General; those for the United States are computed as follows:—

Age Distribution,* England and Wales.			Death-rates, United States.		Products.	
	1901.		1920.			
Ages.	Males. (*a*).	Females. (*b*).	Males. (*c*).	Females. (*d*).	(*a*) and (*c*). (*e*).	(*b*) and (*d*). (*f*).
0– 5 . .	570	572	21·8	17·5	12,426	10,010
5–15 . .	1,048	1,051	2·15	1·75	2,253	1,839
15–25 . .	947	1,011	3·85	3·75	3,646	3,791
25–35 . .	764	852	4·9	4·8	3,744	4,090
35–45 . .	594	635	7·8	6·9	4,633	4,381
45–55 . .	429	463	13·2	11·4	5,663	5,278
55–65 . .	279	318	27·0	23·2	7,533	7,378
65–75 . .	147	184	61·5	53·5	9,040	9,844
75 and over .	57	79	145·7	137·2	8,305	10,839
	4,835	5,165	—	—	57,243	57,450
	10,000					

The standardized rates are, then: Males, $(e) \div (a) = 11{\cdot}8$, and Females, $(f) \div (b) = 11{\cdot}1$. All $\{(e) + (f)\} \div 10{,}000 = 11{\cdot}5$.

The standardized rates in England and Wales in 1924–6 and in the United States in 1920 are lower than the "crude" or recorded rates, because these populations included so much a larger proportion of the elderly than in England and Wales in 1901 as to outweigh the smaller proportion of the very young.

It is noticeable that the standardized rates in England and Wales have fallen more rapidly than the crude rates, and the

* The figures on p. 102 are abbreviated from these.

importance of the modification is emphasized by the fact that the standardized rates are lower in England and Wales in 1924–6 than in the United States in 1920, though the crude rates are higher.

A similar process is necessary in comparing birth-rates and marriage-rates, which also evidently depend on the sex and age distribution of the population. As alternatives to complete standardization, birth-rates are often reckoned per 1,000 women aged 15 to 45, and marriage-rates per 1,000 persons of marriageable age.

Particular attention is given to the death-rates of infants, and for this purpose a special quotient, termed *infant mortality*, is formed, in which the number of deaths in a year of infants under one year old is divided by the number of thousands of infants born alive in that year. Infant mortality has diminished very rapidly in recent years, the diminution affording some compensation for the fall in the birth-rate.

INFANT MORTALITY

Deaths per 1,000 births.

		England and Wales.			
1871–80	149	1908	120	1918	97
1881–90	142	1909	109	1919	89
1891–95	151	1910	105	1920	80
1896–1900	156	1911	130	1921	83
1901	151	1912	95	1922	77
1902	133	1913	108	1923	69
1903	132	1914	105	1924	75
1904	145	1915	110	1925	75
1905	128	1916	91	1926	70
1906	132	1917	96	1927	70
1907	118				
		United States (Registration Area).			
1917	94	1920	86	1923	77
1918	101	1921	76	1924	71
1919	87	1922	76	1925	72

5. The method of standardizing, correcting or adjusting the death-rate can only be used when the death-rates in conjunction with age grouping are known. This is commonly

not the case (except at best for a country as a whole), except at Census dates, and an alternative method is therefore in use. This consists in establishing a correcting or standardizing factor at the date for which the age grouping is known in the district, and applying this factor to the crude death-rate in the district at other dates.

The method may be explained by applying it to the whole population of England and Wales in 1921, and comparing the result with that obtained by the first method, already used for nearly the same figures.

Denote the numbers in the age groups of the standard population by S_1, S_2 . . ., with the total 1,000, and write the corresponding death-rates as D_1, D_2 . . . Let the age-groups in the other population at the date when they are known be s_1, s_2 . . . per 1,000.

Form the products S_1D_1, S_2D_2 . . . and add them, and the products s_1D_1, s_2D_2 . . . and add them. Then

$$\frac{S_1D_1 + S_2D_2 + \ldots}{1{,}000}$$

is the standard death-rate of the standard population, and $\frac{s_1D_1 + s_2D_2 + \ldots}{1{,}000}$ is the death-rate that would be found in the other population, with its own age grouping, but with standard death-rates. The difference between the two is solely due to difference in age-grouping, and $\frac{S_1D_1 + S_2D_2 + \ldots}{s_1D_1 + s_2D_2 + \ldots}$ is the standardizing or correcting factor, which is assumed to be unchanged in subsequent years.

The standard death-rates for England and Wales in 1901 and in 1921 are then 16·95 and 17·56, and the correcting factor for 1921 is $16{\cdot}95 \div 17{\cdot}56 = {\cdot}966$.

Now the crude death-rate in England and Wales in 1921 was 12·1, and the death-rate standardized by this method is, therefore, $12{\cdot}1 \times {\cdot}966 = 11{\cdot}7$.

The other method applied to the same figures gives 11·3—an unusually great difference.

Ages.	Age distribution, England and Wales. 1901. S.	Age distribution, England and Wales. 1921. s.	Death-rates, England and Wales. 1901. D.	Products. SD.	Products. sD.
		Males.			
0– 5	57	44·5	59·0	3,363	2,625
5–15	105	95	3·15	331	299
15–25	95	84	4·1	389	344
25–35	76	69	6·2	471	428
35–45	59	66	10·6	625	700
45–55	43	56	18·0	774	1,008
55–65	28	36·5	33·5	938	1,223
65–75	15	19·3	67·8	1,017	1,309
75 and over	6	6·7	153·2	919	1,026
Total	484	477	—	8,827	8,962
		Females.			
0– 5	57	43·5	49·5	2,822	2,154
5–15	105	94	3·25	341	306
15–25	101	92	3·5	354	322
25–35	85	83·1	5·3	450	440
35–45	64	75·2	8·7	557	654
45–55	46	60·1	13·8	635	829
55–65	32	40·4	26·5	848	1,071
65–75	18	24·1	56·5	1,017	1,362
75 and over	8	10·6	137·5	1,100	1,457
Total	516	523	—	8,124	8,595
Grand total	1,000	1,000	—	16,951	17,557

The second method can then be applied to subsequent years till there is a new record of age grouping.

The two methods may be compared algebraically. Using the symbols S, s, D as before, now write d_1, d_2 . . . for the actual death-rates in the second population. Then

$$\frac{1}{1,000}\Sigma(Sd) \qquad \frac{1}{1,000}\Sigma(sD)$$

is the crude or recorded death-rate, when Σ denotes summation.

In the first method the standardized death-rate is simply $\frac{\Sigma(Sd)}{1,000}$.

In the second the standardizing factor is $\frac{\Sigma(\mathrm{SD})}{\Sigma(s\mathrm{D})}$, and the corrected death-rate is $\frac{\Sigma(sd)}{1{,}000} \times \frac{\Sigma(\mathrm{SD})}{\Sigma(s\mathrm{D})}$.

Both may be written as the crude death-rate multiplied by weighted averages of $\frac{\mathrm{S}}{s}$. For the first equals

$$1{,}000 \times$$

and the second equals $\frac{\Sigma(sw)}{1{,}000} \times \frac{\Sigma(\mathrm{SD})}{\Sigma(s\mathrm{D})}$. Hence by the principles of weighted average the results may in general be expected to agree closely.

The second method used to be applied to the principal towns in England and Wales. Thus for London and Plymouth in 1901 the correction-factors were 1·0656 and ·9720 (based on England and Wales age-distribution in 1901). The crude death-rates were 17·63 and 17·89, and therefore the corrected rates were 18·79 and 17·39, so that the order of the two towns was reversed. London has relatively fewer children and aged.

Correction factors are worked out in the United States on the basis of the year 1920.

	Correcting factors.	Crude rates. 1920.	Crude rates. 1925.	Adjusted rates. 1920.	Adjusted rates. 1925.
New York . . .	1·108	13·0	12·2	14·4	13·5
Boston .	1·005	15·4	14·8	15·5	14·9
Philadelphia	1·015	14·4	13·2	14·6	13·4
Chicago .	1·090	12·8	11·5	13·9	12·5
San Francisco	·992	14·2	14·3	13·1	13·2

It will be seen that the correction affects the order of the cities in this respect.

6. The importance to public officials of the study of comparative death-rates can hardly be over-estimated. If the death-rate in a district is above that in similar districts there is *a priori* something wrong, and very careful analysis is

needed to determine what it is. Death-rates depend not only on age and sex, whose effect can be tested as in the previous paragraph, but on occupation, as to which statistics are given once in ten years by the Registrar-General, and on occupation combined with age; death-rates are, of course, influenced also by epidemics and by catastrophes, and the years affected in such ways must be ruled out of comparison. One of the most important subjects for study at the present time is infantile mortality, which may be regarded as of such a distinct character from general mortality that the latter should be restricted to the rate per population over five years. There is no doubt that a great part of infantile mortality can be avoided; in considering its magnitude attention should be directed to the age (in weeks and months) of the infant, to the economic position of the parents, to the cause of death, with special reference to obviously avoidable causes and to the annual epidemic of summer diarrhœa, and to the effect on the rate of the presence in the district of workhouses, hospitals and other institutions, where the presence of specially feeble infants may in some cases be expected.

7. Problems relating to sickness and mortality naturally come within the province of medical officers of health, and in many districts these officers present admirable annual reports, tackling the questions of most importance in their localities with statistical and professional skill. It will perhaps be useful to indicate the application of the methods sketched in Part I above to this class of problems.

The most important method is that of *averages* in the form of rates. Besides death-rates, etc., we have the "morbidity-rate" (or "attack-rate") which is the number of cases of a particular disease (multiplied by 1,000 or some other round number) divided by the population, and the "case fatality" rate, which is the number of deaths due to a disease divided by the number of cases. Here, as with death- and birth-rates, the denominator must be chosen carefully; for the morbidity-rate the persons should be grouped by ages, districts, etc., so that the classes with different degrees of liability to the

particular disease shall be considered separately. For the "case fatality" rate, great care must be taken to include all the cases, and to be certain of the diagnosis. If differences of treatment (hospital or home) or the efficacy of protection (vaccination, isolation, etc.) are in question, there is always the risk that the ages or economic conditions of the classes considered may differ, and the groups must be made similar before comparison is attempted.

All through vital statistics there is great risk of inadequacy of, and even of mistakes in, *definition*. These arise (i) from intrinsic difficulty of classification and incomplete standardization of description; (ii) from unconscious personal bias of the practitioner; (iii) from the presence of two diseases together, or a disease and an accident; (iv) from the desire to avoid the statement of the existence of certain classes of disease (*e.g.* alcoholism). The presence of any of these may affect the apparent death-rate from any cause, and also the morbidity and case-fatality rate.

In considering questions of cause and effect, liability of various classes, and results of different treatments, the essential thing is to get the exact difference to be considered clearly stated, and then to proceed to analysis by *tabulation*. If the headings of the table prove to be clear and distinct and to follow the differences needed in the problem, the table is good and relevant. Tabulation, when it is not analysis should either be omitted to save space if quite unimportant, or relegated to an appendix if the data may be wanted at some other time, or fitted into standardized tables in a statistical section if they are needed for comparison. The main line of argument or of information should not be interrupted by tables which do not give definite answers to definite questions.

Accuracy.—There is very grave risk in most vital statistics of spurious accuracy. In the practical question whether one rate is greater than another, after the classes concerned have been made similar there remains natural variation; if all known circumstances were the same, differences would

still be found. All records of births, deaths, marriages, sickness, must be regarded as *samples*; the greater the number of persons considered the more accurate the average obtained from the samples. The only non-mathematical test of this accuracy is the test of subdivision (see Chapter VII above), that is, the finding the amount of agreement if smaller groups are taken; the mathematical tests are extremely important, but should only be used when thoroughly comprehended, and are therefore not summarized here. A very great number of differences that are remarked on, prove on mathematical examination to be only the result of chance variation, and to be no more remarkable than (say) the throwing of double-six twice in succession. Here we can only recommend extreme caution in drawing conclusions.

As a simple and obvious rule, based on the elementary ideas of accuracy (Chapter II, above), the rate should never be reckoned to more digits than there are in the numerator (number of cases, etc.).

Diagrams should be used sparingly and with reference to the methods discussed in Chapter V above; they are often specially useful in tracing the course of an epidemic, and in the relation of the seasons to the incidence of some diseases. (See *Studies in Statistics*, Dr. Longstaff.)

8. It is often remarked, and has great theoretic and practical interest, that averages arising from apparently quite fortuitous causes are nearly unchanged from date to date. The death-rate attributed to "varicose veins" in England and Wales was between 2·1 and 3·7 per *million* persons living every year from 1875 to 1894; similarly the annual rate for "accident or negligence" was in the same period 703, 662, 632, 667, 602, 589, 608, 583, 592, 567, 549, 540, 558, 528, 528, 565, 574, 553, 576, 537, a series of small variation with a downward trend. It is this partial constancy in the total of events based on very large numbers which makes insurance possible. In these instances the events are nearly independent of each other; as a contrast notice the death-rates when the events are not independent, owing to infection, or to fashion

in diagnosis; *e.g.* Influenza, 1875 to 1894 : 19, 8, 8, 8, 10, 7, 4, 3, 4, 3, 5, 3, 3, 3, 2, 157, 574, 534, 325, 220.

It is when we obtain approximate constancy or a trend with small variation over a series of observations, as in the case of the general birth-, death- and marriage-rates, and the distribution by sex and by age, that we can apply statistical methods for the elucidation of problems and the tracing of cause and effect.

NOTE.—Administrative were substituted for registration areas in the Registrar-General's Annual Reports from 1911 onwards.

CHAPTER III

TRADE AND TRANSPORT

1. THE statistics of the External Trade of the United Kingdom are published as follows :—

Early in every *month* a cheap unbound account is issued stating the quantity and value of the exports and imports of each commodity, showing the principal sources and destinations of each, with figures totalled for the months of the current year, and comparative statistics for the two previous years. Home produce is separated from foreign and colonial. Accounts of the movement of bullion and of shipping are also included. The details in this monthly issue are subject to correction.

A bulky *Annual Statement of the Trade of the United Kingdom* is issued in four volumes, in which statistics for five years are given. That containing the figures for 1922–5 was published in 1927; Volume I contains details of commodities imported and re-exported; in Volume II these are classified by the countries from which they come. Volume III gives similar statistics for Exports of British Produce, classified by countries. In Volume IV is shown the detailed trade with each country, and also the principal exports and imports at each port. A separate volume deals with *Navigation and Shipping*.

The Statistical Abstract for the United Kingdom, issued in August, summarizes all the statistics of trade and shipping and gives considerable detail as to commodities, but does not show commodities in relation to countries (except for cereals, cotton and wool) for which Volumes II, III and IV of the Annual Statement are the only sources.

2. The basis of these returns is as follows: The exporter of goods or his agent is bound to send a statement of the quantity and value of the goods he is exporting to the proper customs officer, who in general accepts the statement; but every bale, etc., on board ship has to be accounted for before the ship is "cleared," *i. e.* permitted to leave the port.

All imports have to be passed through a custom-house; the importer or his agent hands a statement of the goods he desires to have passed, and the customs officers examine the goods with sufficient care to assess duty, if any, or to verify the absence of dutiable goods. These officials check the values, from current price lists or otherwise, and insist on the furnishing of the requisite details. Returns of the values of imports and exports are further checked at the Central Customs Statistical Office, and inquiry is made if the entries appear unusual or are incomplete.

In this process there is a good deal of room for inaccuracy in detail, which may be important for special classes of goods; but there seems no reason to doubt that the descriptions and quantities are stated on the whole with fair accuracy. The values are often a matter of estimate (*e. g.* in the case of goods exported for sale by a foreign agent), and our only security for accuracy is that in a composite total (see p. 29 above) errors which are not biassed tend to neutralize one another, and that, though there are inducements in some cases to exaggerate value, there are inducements in other cases to under-value.

In the case of exports the value is intended to be that of the goods after all internal transport and dock expenses are paid, that is the value at which the goods are delivered free-on-board (f. o. b.). For imports the value is intended to be that of the goods before they are landed, and includes their cost, insurance and freight (c. i. f.). Thus exports are valued at the moment they pass out of the hands of British shore-labour, and imports before they are handled or pay duty. If the exchange were simply across a land frontier, and the goods of one community were exchanged as a whole against the

goods of another, it is clear that the method described would give equal values for imports and exports.

As a matter of fact goods are often quoted at prices to include delivery; in these cases the value has to be corrected for the trade statistics.

3. The following table gives the total trade statistics for 1925.

UNITED KINGDOM

	000,000's. £		000,000's. £
Imports of Merchandise	1,321 A	Exports of Produce of the United Kingdom	773 B
Imports of Bullion, etc.	52	Exports of Imported Merchandise . .	154
Total Imports .	£1,373	Total Exports of Merchandise . . .	927 C
		Exports of Bullion, etc.	62
		Total Exports .	£989

Transhipments under bond £29 Mn. G.

The total A is always quoted as the value of imports, and B is generally quoted as that of exports.

Goods landed may be transhipped either at the same or another port under bond, that is, without passing out of the control of the customs officials, in which case they are entered as "Transhipment" (G) and not included in imports and exports. Goods which pass out of control of the customs are either for use or consumption in the United Kingdom, or for sale again in another country; all such are counted as imports, but when imported goods come to be re-exported they are declared as of foreign or colonial origin. The value A of imports is then thus composed—

MERCHANDISE ONLY

	000,000's.
Imports for consumption	1,167 D
Imports for re-exportation . . .	154 E
Total	£1,321

Since the goods are valued afresh for exportation, they are

presumably increased in value by the expense of handling them in the country, and the value E is thus too great.

" A " should be compared with C, and D with B.

Actually no theoretic line can be drawn between goods which are (i) simply transhipped, (ii) goods which are re-exported unchanged, (iii) goods which undergo some slight alteration and are re-exported, (iv) imported goods which form some constituent part of a machine which is exported, (v) imported yarn which is exported when woven, (vi) imported wool which is spun and woven and then exported. (i) is included in neither exports nor imports, (ii) is included in imports and in exports of foreign produce, (iii) to (vi) are included in imports and in exports of produce of the United Kingdom. It is not possible to correct this method, but it is important to understand it and consider it in the light of pp. 64–5 above.

Notice that bullion and specie, *i. e.* metallic and coined gold and silver, enter nearly equally on both sides of the account.

No special record is kept of trade from one port to another of the United Kingdom or of islands in the British Seas, but, from 1923, the trade of the present United Kingdom with Southern Ireland is shown separately.

4. Other countries have different methods of definition, valuation and classification.* Before using their statistics, it must be ascertained how imports for consumption, for re-exportation with or without alteration, and exports of national and foreign produce are treated, whether bullion and specie are included, exactly what districts are included in the country concerned, and whether there are any peculiarities in the method of valuation.

A general method is as follows: Goods are valued with the intention of producing results on the basis described

* See Reports of the Committee of the British Association on " The Accuracy and Comparability of British and Foreign Statistics of International Trade," 1904 and 1905, and Memorandum 21 of the *London and Cambridge Economic Service.*

above for the United Kingdom. Bullion and specie are excluded. All goods entering and leaving the country are included in totals of *General* Imports and Exports; goods for consumption or use in the country and exports of goods which have been produced or undergone any process of manufacture in the country are included in totals of *Special* Imports and Exports. General exports are thus greater than special exports by the value of goods passed in and out of or through the country, and similarly with imports; the differences for exports and for imports are approximately equal.

For the United Kingdom we should have in 1925.

		000,000's.
General exports . .	C + G	£956
Special exports . .	B	£773
General imports . .	A + G	£1,350
Special imports (approx.)	D	£1,167

It should be noted that the United States and Canada and British South Africa value imports, not on arrival at the port of destination, as is general, but at the place of manufacture.

5. If we regard the international trade of the world as a whole, a consignment forming part of the special exports of one country may appear under general imports and exports of all the countries it passes through, but will finish as a part of the special imports of some one country. The same consignment will be worth more as imports than it was as exports by the cost of transport (including freight, insurance, transhipment and commissions). The table on p. 125 shows the relation of the special imports and exports of the principal trading countries of the world for the year 1924. The numbers given are subject to many minute corrections; after these are made it is found that imports on the whole are worth about $6\frac{1}{3}\%$ more than exports as a whole.*

* The $6\frac{1}{3}\%$ is obtained as follows. Subtract from exports $287 Mn., value of bullion, etc., exported from South Africa and some other countries, and not included anywhere as imports. Add 5% to the imports of Australia and New Zealand, who it is considered make insufficient allowance for freight. We have then imports $29,017 Mn.,

A similar calculation made for 1912 showed an excess of 13%. Both estimates are very rough, but the diminution marks the fall in freights relative to the value of the goods carried; freight-rates, in fact, from 1912 to 1924 did not rise so rapidly as the price of commodities (see pp. 140 and 144 below). In 1912 the excess value of imports over exports was about £450 Mn., in a review of rather more than 30 countries, which included the bulk of the world's trade. In 1924 the excess was about $1,750 Mn., or £395 Mn. at the then rate of exchange, all the trading countries being included. This difference is received by those engaged in any capacity in international transport, and of it a large share appertains to the citizen of the United Kingdom. The table is compiled from a publication of the League of Nations.

6. The balance of trade between the United Kingdom and the rest of the world is composed of many categories of payments. The most important of these are for Imports and for Exports of merchandise; except in the gold-producing countries and India the balance of bullion is generally small. Next comes the interest due on capital invested overseas and for short-term loans, and on the other side of the account new investments, which in turn will yield interest in subsequent years. Thirdly, we have to include shipping services. There are many other items, such as fire-insurance, financial services and commissions, payments of foreign branches of firms to headquarters, remittances from emigrants, payments by foreign visitors, etc., which enter into the balance, but for which there can be no accurate account. In all such cases there are visible exports from one country recorded also as imports by another (so that the world's balance as discussed in the previous paragraph is not affected), but

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exports $27,565 Mn. Add $x\%$ to the imports of U.S.A., Canada and South Africa, where $x\%$ is the sought value for freights. Then we have the equation

$$27{,}565\left(1 + \frac{x}{100}\right) = 29{,}017 + 4{,}654 \times \frac{x}{100}$$

of which the solution is $x = 6{\cdot}3$.

INTERNATIONAL TRADE, 1924

Special Imports and Special Exports of Merchandise, expressed in American dollars at current rates of exchange.

Unit $000,000

	Imports.	Exports	Excess of Imports.	Excess of Exports.
Norway	211	145	66	—
Sweden	378	335	43	—
Denmark	371	331	40	—
Germany	2,162	1,559	603	—
Poland	285	244	41	—
Holland	901	667	234	—
Belgium	815	644	171	—
Switzerland	452	364	88	—
Spain	568	346	222	—
Italy	844	626	218	—
Austria	486	278	208	—
Hungary	143	117	26	—
Greece	144	59	85	—
Algeria	167	115	52	—
United Kingdom	5,022	3,538	1,484	—
South Ireland	301	221	80	—
Australia	642	629	13	—
China	825	625	200	—
Japan	995	728	267	—
Turkey	194	120	74	—
Russia	222	268	—	46
Finland	117	125	—	8
France	2,088	2,169	—	81
Czechoslovakia	469	502	—	33
Jugo-Slavia	106	122	—	16
Roumania	131	139	—	8
Egypt	217	296	—	79
India and Ceylon	847	1,312	—	465
Dutch E. Indies	262	576	—	314
British Malaya	335	369	—	34
Philippines	108	135	—	27
Korea	129	136	—	7
British S. Africa	290	372	—	82
New Zealand	214	227	—	13
Canada	789	1,070	—	281
United States	3,575	4,498	—	923
Mexico	156	298	—	142
Cuba	288	435	—	147
Brazil	302	420	—	118
Argentina	650	793	—	143
Chile	120	211	—	91
Other Countries	1,653	1,688	—	35
	28,974	27,852	1,122	—

there is no visible trade in the opposite direction. Rough estimates of the amounts due are made by the Board of Trade every year, and the following example is compiled from its statistics.

BALANCE OF TRADE OF THE UNITED KINGDOM IN 1927

Due to U.K. for—		Due from U.K. for—	
Exports of Merchandise . .	£832Mn.	Imports of Merchandise .	£1,219Mn.
Exports of Bullion, etc. . . .	36	Imports of Bullion, etc. .	41
Income from Investments * .	270		
Shipping Services *	140		
Other Services * .	78		
	£1,356		£1,260 †

Balance in favour of the United Kingdom, £96 Mn.

This balance was, in fact, available for increase of investment abroad in the year.

7. The table on page 127 shows in some detail the values of imports and exports since 1855; the statistics of imports prior to 1855 were not computed on the same basis. To follow the history of the external trade as a whole, smoothed diagrams (the averages being taken over eight or more years), should be constructed as on pp. 40–2 above. It will then be seen that there has been a general but not uniform upward trend throughout the period till 1913, concealed or accentuated by considerable fluctuations.

8. The fluctuations both of imports and of exports are principally due to movements of price, and unless we eliminate these we obtain a very imperfect view of the course of trade. The following chapter shows how considerable these movements have been. The method generally used for studying the quantity, or volume, of trade, as distinguished from its value, is as follows: The prices of all goods for which definite

* Net: *i. e.* sums due to U.K. less sums due for similar services due to other countries.

† Payments on Government account to the United States for interest are assumed to be balanced by receipts from reparations and certain other items.

External Trade of the United Kingdom (000000's)

	Imports, less re-exports.		Exports of Home Produce.	
	Declared value.	Estimated value at prices of 1902.	Declared value.	Estimated value at prices of 1902.
1870	£259	£160	£200	£142
1871	271	165	223	150
1872	296	185	256	162
1873	315	195	255	160
1874	312	200	240	160
1875	316	200	223	159
1876	319	223	201	155
1877	341	230	199	159
1878	316	231	193	160
1879	306	234	191	171
1880	348	254	223	194
1881	334	244	234	211
1882	348	258	241	211
1883	361	275	240	217
1884	327	268	233	220
1885	313	272	213	211
1886	294	270	213	222
1887	303	283	222	231
1888	323	294	234	242
1889	361	326	249	251
1890	356	324	263	250
1891	373	339	247	235
1892	359	339	227	227
1893	346	336	218	222
1894	350	364	216	230
1895	357	384	226	249
1896	385	410	240	261
1897	390	415	234	257
1898	410	437	233	256
1899	420	437	264 (255)*	272
1900	460	442	291 (284)	262
1901	454	454	280 (271)	267
1902	463	463	283 (278)	283
1903	473	469	291 (286)	291
1904	481	477	301 (296)	301
1905	487	473	330 (324)	327
1906	523	489	376 (367)	358
1907	554	500	426 (416)	380
1908	518	484	377 (366)	352
1909	533	494	378 (372)	368
1910	574	503	430 (422)	406
1911	577	520	454 (448)	418
1912	633	560	487 (480)	439
1913	659	580	525 (514)	455
1920	1,710	510	1,334	322
1921	979	427	703	227
1922	899	502	720	313
1923	978	565	767	350
1924	1,137	616	801	364
1925	1,167	634	773	364
1926	1,117	690	653	325
1927	1,096	708	709	375

* The numbers in brackets exclude the value of ships built at home and sold to foreigners, which was not ascertained or included prior to 1899.

quotations can be made are ascertained for a particular year or short period; the quantities of goods exported or imported are then valued in each separate year at these standard prices; it is then assumed that the differences in value shown for the goods which can be priced are typical for all goods. *E. g.* to take an imaginary example—

Value of imports in (say) 1890, as stated in the accounts, *i. e.* at the prices of 1890, £356 (millions). Take 1902 as year of standard price. Suppose that £300 worth of the 1890 imports can be separately valued, and are found to be worth £273 at 1902 prices, the prices in 1890 being higher than those in 1902; then it is assumed that the whole £356 would be reduced in the same ratio, viz. to £356 $\times \frac{273}{300} =$ £324, if all could have been valued.

Such a calculation was carried out over a long period by the *Economist* newspaper, goods each year being valued at the prices of the year before. Since about 1905 the Board of Trade has made similar calculations. From these and other sources rough estimates have been made as in the table, with the prices of 1902 as a basis.

The method is open to a good deal of criticism in detail, but there is no doubt that it leads to results that are substantially correct, at any rate over short periods.

It is interesting to notice how small, before the War, were the actual fluctuations in quantity, as indicated by the values at unchanged prices, especially in imports. Consumption of goods and, to a very great extent, production went on with little change in times of commercial inflation and depression.

Since the War the very violent price movements and the unsettlement of trade have resulted in much greater fluctuations than before; but still, except in the year of depression 1921, and during the coal-stoppage of 1926, there has been fair regularity in the movement when price changes are eliminated.

The necessity of some such examination is emphasized by the consideration that a rise of 1*d.* per lb. in the price of

raw cotton would have raised the value of imports by about £7,000,000 in 1925, and since perhaps three-quarters of the cotton manufacture is for export, the value of exports is also raised by over £5,000,000: these immense changes would take place without any change in quantity or in the work done by British capital and labour.

9. The tabulation of the statistics of the foreign trade was greatly improved in 1904, and the new method was carried back to 1891 in the Statistical Abstract for 1905. The table on p. 130 shows the statistics in the principal categories.

The complete meaning of the classification can only be seen by looking at the detailed list in the Monthly or Annual Trade Accounts; but it may be mentioned that commodities such as yarn and pig-iron, which are the finished product of one process and the raw material of another, are classed as "mainly manufactured." *

In the lower part of the table are shown the values for coal, the principal exported raw material, and of the principal groups of manufactures.

In using the table it must be remembered that 1900 and 1907 were years of exceptionally high prices.

10. The original sources of imports and ultimate destinations of exports cannot always be known. If, for example, wool grown in Turkey were spun in Austria, woven in Germany, sent by rail through Holland, manufactured into ready-made clothes in Leeds, and sold in Canada, it would figure in the export and import statistics of many countries, and its value would be due to the co-operation of many nations. Again, if goods are sent from London to Antwerp for sale, they may pass on to Germany, Russia, Austria or Switzerland without the English manufacturer knowing their destination. Before 1904 imports were only stated as from the country from which they were last shipped, while exports have been stated as to the country of ultimate destination, as a rule, since

* A more detailed examination of classes of manufactured goods is given in Cd. 2337, Mem. xii, and continued in Cd. 4954, pp. 48 *seq.*

Classified Values of Imports and Exports of the United Kingdom (000,000's)

Imports, whether for consumption or export.*	Averages. 1891–1897	1898–1902	1903–1905	1906–1908	1909–1911	1912–1913	1919‡ Old	1919‡ New	Years. 1920	1921	1922	1923	1924	1925	1926	1927
Food, Drink and Tobacco	£185	217	231	243	259	285	707	719	766	567	472	509	571	570	530	539
Raw Materials and Articles mainly unmanufactured	143	161	181	219	243	279	646	607	710	271	298	325	400	425	392	352
Articles wholly or mainly manufactured	96	125	139	151	157	190	267	295	453	244	230	257	300	320	315	322
Unclassified	2	3	2	2	3	3		6	4	3	3	5	6	6	4	6
Total	£426	506	553	615	661	756			1,626	1,933	1,086	1,003	1,096	1,277	1,321	1,241	1,219
Exports of home produce.																
Food, Drink and Tobacco	£11	14	17	22	26	32	33	34	51	37	36	44	57	55	50	52
Raw Materials and Articles mainly unmanufactured	22	34	36	50	53	64	121	111	146	64	102	131	106	84	47	76
Articles wholly or mainly manufactured	196	218	249	314	334	398	632	641	1,120	589	569	580	619	617	539	564
Unclassified	2	4	5	6	8	11		13	17	13	13	12	19	17	17	17
Total	£230†	270	307	393	421	505		799	1,334	703	720	767	801	773	653	709
Selected Exports.																
Coal and Fuel	£16	28	27	35	38	48		83	100	43	73	100	72	50	19	46
Iron and Steel Manufactures	22	27	30	41	42	51		64	129	64	61	76	74	68	55	69
Machinery	15	19	21	30	29	35		31	63	75	52	45	45	49	46	50
Ships†	—†	8	5	10	7	9		2	27	31	30	10	6	6	5	5
Cotton Manufactures	66	70	83	102	106	125		239	401	179	187	177	199	199	154	149
Wool „	25	23	27	31	35	38		96	135	55	58	63	68	59	51	57

* Excluding transhipment under bond.

† Ships were not included as exports prior to 1899.

‡ The classification was revised in 1919–20. The principal changes were the transference of feeding-stuffs for animals from manufactures to food, and of refined oils (petrol, etc.) from materials to manufactures.

NOTE.—Since each entry and also each total is given to the nearest £1,000,000, the sum of the items in some cases differ by 1 from the entry for the corresponding total.

1894.* Thus Switzerland, Bolivia and Rhodesia,† which have no seaboard, had no place in our statistics. German and Russian goods were entered as imported from Holland, Austrian and Swiss goods from Belgium, and so on. From 1904, a second method has been used, and the tabulation on the former plan was first relegated to a supplemental volume of the Annual Report, and then given up.‡ Importers have stated the country from which goods are actually *consigned* to them; this is generally also the country in which they were produced or manufactured or received their last process of manufacture; exporters have also stated the countries to which goods were *consigned*, which are in general the ultimate destination. The following short table shows the results for certain European countries. The imports in the second column are in a very different proportion from that in the first ; the exports generally do not differ in the third significant figure.

	Imports 1907.			Exports 1907 (including Foreign and Colonial produce).	
Received direct from		Consigned from	Consignments retained for consumption.	Exported to	Consigned to
	00000's				
Russia	£314	329	306	191	191
Germany	388	572	541	567	567
Holland	368	160	154	190	190
Belgium	283	175	168	194	169
France	528	463	398	335	332
Austria	11	68	64	54	54
Switzerland	0	84	72	0	29
	£1892	1851	1703	1531	1532

* Prior to 1894 they were credited to the country to which they were shipped direct.

† In these cases exports were credited to the port of landing.

‡ The Statistical Abstract for 1907 shows the results as in the table here given. That for 1914 gives the countries of shipment in Table 33. and of consignments in Tables 34, 35. From 1915 only consignments are given and the Tables are rearranged. The monthly accounts now give countries of consignment, not of shipment.

11. Statistics relating to the trade of the United States can be studied in the same way as those of the United Kingdom. Prior to 1916 the principal tables related to the fiscal year ending June 30th; from 1916 the calendar year has been used. Imports are valued at the place of origin.

Recent statistics are as follows :—

FOREIGN COMMERCE OF THE UNITED STATES
(Merchandise, 000,000's).

	Imports.		Exports of Domestic Produce.	
	Declared value.	Estimated value at price of 1913.	Declared value.	Estimated value at price of 1913.
1913	$1910	$1910	$2540	$2540
1919	4130	2370	8060	3610
1920	5670	2660	8440	3480
1921	2670	2280	4620	3050
1922	3280	2900	3960	2840
1923	4010	3040	4330	2920
1924	3830	2970	4760	3300
1925	4460	3150	5080	3480
1926	4670	3360	4990	3710

NOTE.—The Index Numbers (U.S. Stat. Abs., 1926, p. 441) do not allow accuracy in the last integer, which must be regarded as only approximate.

It is interesting to compare the accounts of what presumably is the same trade as stated by two countries concerned.

U.K. Accounts.	1925. (000,000's)	U.S. Accounts.
Exports to U.S. :		Imports from U.K. :
Home produce . .	£52·1	$413 = £85·5
Re-exports	31·1	
Imports from U.S		Exports to U.K. :
Retained .	£228·6	$1,034 = £214
Re-exported	16·7	

$ are converted at the rate 4·83, the average for 1925.

It appears that the United States counts as coming from the United Kingdom all goods sent, whether produced in the United Kingdom or re-exported. The imports into the United Kingdom are naturally greater in value on arrival than on leaving the United States, since the shipping cost is included.

12. Both imports into the United Kingdom, as a whole, and exports from the United States vary greatly according to the time of the year. Among the former, cotton, wool, wheat and timber are specially marked in this respect, and, of course, the United States is not the only country of origin, even for cotton.

SEASONAL VARIATION * (based on the years 1905–13)

Percentage by which the value in each month is greater or less than one-twelfth of the value in a year.

	All Imports into U.K.	All Exports from U.S.		All Imports into U.K.	All Exports from U.S.
January .	+ 9	+11	July . .	− 7	−24
February .	− 2	− 4	August .	− 9	−17
March . .	+ 4	− 3	September .	− 8	0
April . .	− 2	− 7	October .	+ 6	+23
May . .	− 4	−12	November .	+10	+24
June . .	− 9	−18	December .	+12	+27

The February trade is diminished by the shortness of the month; if this is allowed for, the entries for it would be approximately + 5 and + 3, and the whole movement becomes nearly continuous from a minimum in June or July to a maximum in December.

13. Shipping statistics call for little comment except as to the meaning of tonnage (see note at the end of the chapter). The Statistical Abstract gives a series of useful and easily intelligible tables on the subject. Every ship is registered as of a definite nationality, which is generally that of her owners, but, for example, the United States may have considerable holdings in ships in the Atlantic trade registered as British. On *entering* a port of the United Kingdom the ship's papers must be shown, stating whence she came, where she last broke bulk, what cargo she carries, and her register tonnage. She cannot *clear* from the port till her papers have again been seen, her next destination stated, and the necessary declarations of her cargo are in order. In theory, nothing

* See Memorandum No. 7 of the *London and Cambridge Economic Service*. The work for the table in the text is only rough.

enters or leaves the United Kingdom without official knowledge.

Steam- and sailing-ships are distinguished. It must be remembered that a steamship carries much more cargo between the same two countries in a year than a sailing-ship of the same carrying power, owing to her greater speed and more frequent journeys. Coastwise and foreign voyages are distinguished; coastwise means between any two ports in the United Kingdom or islands in the British seas; foreign is from or to a port in the United Kingdom to or from a port in a foreign country or one of the British possessions. Since April 1923 Southern Ireland is counted as foreign.

For separate ports entrances and clearances are no longer published, but all vessels that enter the port for any purpose other than shelter are counted as "arrived" and subsequently as "departed." Thus we have such statistics as the following.*

CARDIFF, 1925

(Cargoes or Ballast. Tons 000's)

	Arrived.	Departed.
In Foreign Trade . . .	5,497	7,293
„ Coasting Trade . . .	2,969	1,266
	8,466	8,559
Value of Imports . . .	£37·3 Mn.	
„ of Exports . . .	10·3 „	

None of the tables show the aggregate of the voyages of British ships or any other measure of the work done by them, and it must be realized that some part of the merchant navy carries cargo between distant ports without calling at home at all.

The following table from the Annual Report of the Chamber of Shipping of the United Kingdom, shows the tonnage of shipping registered under various flags, distinguishing all for which the total was over 1,000,000 tons. Thus, using the earlier tables in this chapter, we find that *circa* 1926,

* *LXXth Statistical Abstract for the United Kingdom*, pp. 265, 267, 271, 273, 299.

32,500 ships, with aggregate tonnage 64,700,000, carried exports to the value of about £6,000 Mn. per annum (together with the inland trade on North American Lakes). Per ton of shipping this is about £94 per annum, and if we reckon freight, as above computed, at $6\frac{1}{3}\%$, we get that shipping earnings average about £6 per ton per annum, or rather more since the divisor includes ships idle throughout the year. On this basis the earnings of British ships would be 19·4 Mn. × £6 = £116 Mn., which is less than the £140 Mn. estimated in the table on p. 126, which, however, includes other items besides freights.

STEAM AND SAILING TONNAGE ON LLOYD'S REGISTER
(Vessels of 100 tons gross and over).

	June 1914.		June 1926.	
	Steam and Motor, gross tons. 000's.	Sailing, net tons. 000's.	Steam and Motor, gross tons. 000's.	Sailing, net tons. 000's.
U.K.	18,892	365	19,264	136
British Dominions	1,632	157	2,689	182
German	5,135	325	3,062	49
U.S. Sea	2,069	946	11,472	973
Lakes	2,260	92	2,348	85
Norwegian	1,957	547	2,807	35
French	1,922	397	3,324	166
Japanese	1,708	—	3,968	—
Dutch	1,472	25	2,553	12
Italian	1,430	238	3,150	90
Austro-Hungarian	1,052	3	—	—
Swedish	1,015	103	1,295	44
Russian	852	202	—	—
Spanish	884	15	1,126	37
Danish	770	50	1,049	32
Others	2,354	221	4,565	271
Total	45,404	3,686	62,672	2,112
No. of ships	24,444	6,392	29,092	3,523

14. Railway statistics used to be deficient in the extreme for the United Kingdom. In the Statistical Abstract the principal known facts were summarized. Apart from finance, these were the length of line open (distinguishing single from double or more), the number of passengers, and the weight of minerals and of other merchandise carried. There was no

information as to the average or aggregate distance travelled (passenger-miles, and ton-miles). The totals were so crude and heterogeneous as to be practically valueless (see p. 65 above), except that over a very few years the total weight carried gives some indication of the upward and downward movements of trade. But since the War the Ministry of Transport has issued a monthly report on Traffic, which gives details of earnings and of operation very fully, so that now we have for each railway-group such information about ton-miles, etc. as is indicated on p. 73 above. Each month statistics are also given of the quantity of selected commodities carried, and generally there is furnished a mass of important information.

NOTE.—*Shipping tonnage.* The definition and measurement of tonnage are extremely complicated, as may be seen from the Report on the Merchant Shipping Bill (H. of C. 256, 1907), where many examples are given. There are at least four measurements of a ship's size or capacity: displacement, dead-weight,* gross register tonnage, and net register tonnage. The displacement is the weight of the ship (unloaded), which equals the weight of the water displaced; the dead-weight represents its weight-carrying capacity; neither of these is used in the general shipping statistics. The gross tonnage is the number of times 100 cubic feet is contained in the ship, measured according to certain rules; 100 cubic feet is taken as representing the space occupied by a ton of cargo, but a ton of coal occupies only about 45 cubic feet, and a ton of water about 35 cubic feet; light or loosely packed cargoes occupy more. Net tonnage is obtained from gross by subtracting according to artificial rules space occupied by the engines (with an allowance for bunker and air space), by the crew's and passenger's quarters and the parts necessary for navigation; the remainder (reckoning as before 100 cubic feet to the ton) is supposed to represent the carrying capacity of the ship, and is the register tonnage. The rules for measure-

* "Burden," nearly obsolete, is equivalent to dead-weight.

ment and deduction differ for different nations, but there has been a wide-spread movement in the direction of adopting the British system. The British rules have been modified from time to time. Actually the register tonnage does not bear any close relation to carrying capacity, and is extremely artificial. All the recent shipping statistics are given in register net tonnage; formerly they were given in gross tonnage, but the present method runs back far enough for all practical purposes, and it is easy in comparative statistics to see if there has been a change in this respect. Sailing-ships' tonnage (which has, of course, no allowance for propelling machinery) should be kept distinct from steamships. Marine architects continually try to build so that the register tonnage shall be as low as possible, since dock dues are charged in proportion to this tonnage, and they take advantage of the rules of measurement so that the deductions allowed shall be as great as possible; in other words, they try to reduce the register tonnage relatively to the carrying capacity. It follows that the growth of shipping tonnage shown in the tables tends to fall short of the real growth of carrying capacity. Further, the Plimsoll mark, which regulates the weight a ship can carry, was raised in some classes in 1906 to allow a greater weight without altering the register tonnage. The general result is that the shipping statistics cannot be used for any fine measurements, and are not comparable over a long series of years.

NOTE.—For transit dues in the Suez Canal, and in the Panama Canal, the register tonnage of all ships is some 15 or 20% higher than that on any national computation, and more nearly approximates to the carrying capacity.

CHAPTER IV

PRICES

1. PRICES from the ordinary commercial standpoint are of course to be found in the trade journals, and summaries from time to time in the *Economist* and the *Statist*. From the statistical point of view we are only concerned with the change in particular prices over a series of years, and with general price movements. For both purposes the most accessible information is to be found in the tables of the Statistical Abstract, which show the prices of exports, imports (prior to 1925), cereals and minerals, and in Mr. Sauerbeck's studies of price movements published annually since 1886 in the *Statistical Journal*. The Board of Trade Report on Wholesale and Retail Prices (H. of C. 321, 1903) contains a great many records of prices over a long series of years, and interesting charts showing the prices of wheat and of bread since 1800 were published in the *Labour Gazette*, May 1909.

2. The great difficulty in the measurement of prices is in the definition of the commodity to be measured. In the case of the staple raw materials of manufacture, cotton, wool, iron, etc., and the principal raw foods, wheat, sugar, etc., the various grades are to a great extent standardized, and it is only after the lapse of a considerable time that difficulties in exact comparisons are felt; for example, wheat prices can be properly compared over (say) 20 years, but in a century the kind of wheat commonly in use has changed immensely. As the raw materials pass through the various stages of manufacture endless varieties are introduced and the goods continually change their character without changing their name, or qualities which were commonly used fell out of fashion;

for these reasons it is not possible to measure the price of such commodities as cotton yarn or cotton piece-goods over a long period; still less can we reckon the change in price of ready-made clothes, of machinery, of bicycles, etc. Similarly the change in character of live stock, of timber, of everything of which the source varies or which can be modified by man, is readily perceptible after even a few years.

3. The measurement of retail prices is so difficult that (except for the more important articles of food) neither government departments nor statisticians have as yet made much progress with it. All the varieties of production and all the changes of fashion have their full influence here. It is seldom that goods can be exactly matched, even in external appearance, after a few years; and a more subtle difficulty is present, for the actual quality of goods is very frequently changed with no corresponding change in price, customers demanding articles at the price they are used to and the manufacturer making slight changes in the constituents to preserve his profit. As an illustration of another difficulty, it may be observed that the price of travelling one mile by railway in the United Kingdom was nominally 1*d.* from the initiation of railways till the Great War, but the kind of accommodation and the speed of travelling have changed completely, and a considerable proportion of the third-class journeys made are in fact charged at a lower rate. We therefore leave the whole problem of retail prices on one side as too complex for the beginner.

4. The Board of Trade prices of imports and exports published, till recently, in the Statistical Abstract, are obtained (after rejecting those commodities, such as pictures, horses, machinery, miscellanea, etc., for which an average price is clearly an absurdity), by dividing the total value of imports or of exports for the year by the total number of units of quantity for each commodity not rejected. The price stated is thus always an average, not a market quotation, and in some cases (*e.g.* carpets and druggets) the divisor is not homogeneous. An apparent change of price is often due to

	1	2	3	4	5	6	7	8	9	10	11	12
	Wheat.	Refined Sugar.	Tea.	Cotton.	Wool.	Jute.	Pig and Puddled Iron.	Coal.	Silver.	Index Numbers.		Sauerbeck's Index Numbers.
										Import.	Export.	
	per cwt. *s.*	per cwt. *s.*	per lb. *d.*	per cwt. £	per lb. *d.*	per cwt. *s.*	per ton. £	per ton. *s.*	per oz. *d.*			
1853–9	12·8	38·2	16·4	3·06	17·3	33·8	3·4	9·3	61·6	155	122	142
1860–4	11·2	34·3	18·4	6·4	16·8	19·6	2·8	9·0	61·3	176	—	146
1865–9	11·8	31·7	19·0	5·7	15·8	17·6	2·9	9·8	60·7	167	—	144
1870–4	12·0	33·8	16·8	3·97	14·3	18·6	4·40	14·4	59·8	160	150	149
1875–9	11·0	30·2	15·8	3·00	14·4	15·0	2·97	10·4	53·6	142	125	131
1880–4	10·2	27·0	12·6	2·91	12·8	14·9	2·74	9·0	51·4	132	111	120
1885	7·8	18·1	12·1	2·86	10·0	11·3	2·18	8·8	48·6	115	101	104
1886	7·5	16·7	11·8	2·49	9·1	11·2	2·16	8·3	45·4	109	96	100
1887	7·6	15·6	10·6	2·51	10·1	11·2	2·36	8·2	44·6	107	96	98
1888	7·7	17·5	11·0	2·59	9·8	12·4	2·13	8·3	42·9	110	97	101
1889	7·7	19·7	10·8	2·64	9·8	14·2	2·51	10·0	42·7	111	99	104
1890	7·8	16·3	10·6	2·67	10·3	13·3	3·05	12·4	47·7	110	106	104
1891	8·9	16·5	10·7	2·59	9·4	12·2	2·63	12·0	45·1	110	105	104
1892	7·7	17·1	10·1	2·39	8·7	15·2	2·57	10·9	39·8	106	100	98
1893	6·4	18·4	9·7	2·43	8·7	13·2	2·35	9·8	35·6	103	98	98
1894	5·3	15·5	9·6	2·06	8·5	13·6	2·30	10·4	28·9	96	94	91
1895	5·5	13·3	9·6	1·94	8·1	11·1	2·40	9·2	29·9	93	91	90
1896	6·2	13·6	9·5	2·31	8·4	12·2	2·39	8·7	30·7	94	92	88
1897	7·4	12·3	9·4	2·09	8·0	11·7	2·41	8·8	27·6	94	91	90
1898	8·0	12·3	9·1	1·80	8·2	10·5	2·63	9·8	26·9	94	91	92
1899	6·7	12·6	8·8	1·91	8·6	12·7	3·47	10·5	27·4	96	97	98
1900	6·8	12·8	8·5	2·61	9·5	14·7	4·20	16·5	28·2	104	111	109
1901	6·6	12·2	7·7	2·57	7·5	13·5	3·13	13·7	27·2	100	105	101
1902	6·7	10·6	7·2	2·54	7·5	12·8	3·24	12·2	24·1	100	100	100
1903	6·8	10·7	7·7	2·80	8·3	13·5	3·15	11·6	24·7	101	100	100
1904	7·0	12·3	7·2	3·13	8·7	13·7	2·92	11·0	26·4	101	100	101
1905	7·2	14·8	7·2	2·65	9·3	17·0	3·15	10·5	27·8	103	101	104
1906	7·0	11·6	7·4	3·09	10·2	22·6	3·50	10·8	30·9	107	105	111
1907	7·7	12·0	8·1	3·31	10·3	22·4	3·71	12·6	30·2	111	112	115
1908	8·4	13·0	8·0	3·03	9·3	16·6	3·17	12·6	24·4	106	107	105
1909	9·3	13·4	8·2	3·09	9·5	15·1	3·24	11·2	23·7	108	103	107
1910	8·4	15·6	8·2	4·07	10·2	15·7	3·42	11·6	24·6	114	106	113
1911	7·9	15·3	9·0	3·61	10·0	19·9	3·22	11·3	24·6	111	109	115
1912	8·5	16·5	8·7	3·20	9·9	21·7	3·69	12·6	28·0	113	112	123
1913	8·3	13·4	9·1	3·64	10·3	26·4	4·30	13·8	27·6	113	116	123
1920	26·9	64·0	15·0	15·0	24·1	60·0	17·26	79·7	61·6	336	416	361
1921	17·5	31·9	12·3	6·93	13·1	36·4	12·37	34·8	36·9	228	311	223
1922	12·2	21·1	14·9	6·69	12·8	28 7	6·55	22·6	34·4	178	225	188
1923	10·2	30·1	17·6	7·90	15·3	27·8	7·20	25·1	31·9	173	215	186
1924	11·8	26·8	19·0	8·48	22·0	30·2	6·87	23·4	34·0	183	216	200
1925	14·0	17·9	18·3	7·32	23·7	42·7	6·19	19·9	32·1	183	208	196
1926	13·1	16·0	18·8	5·34	18·6	44·1	5·23	18·6	28·7	161	197	183
1927	12·3	18·5	18·5	4·78	17·6	30·0	5·92	17·6	25·9	154	185	177

Columns 1–6 are the "Average Import Prices" obtained by dividing the values of the commodities imported, as stated in the Statistical Abstract, by the quantities there stated. Columns 7 and 8 are "Export Prices" obtained in a similar way.

an actual change of quality, and in some cases this change is cumulative, not accidental; for example, if the general run of "heavy broad woollen tissues, all wool," increased in breadth, perfection of manufacture, and finish, it would still be entered under the same category. The table on p. 140 gives examples of prices (wheat, jute, and pig-iron) where the quality has probably not changed much, of others (cotton and coal) where the relative proportions of different qualities have probably changed perceptibly, but not when few years only are considered,* and of others (tea and sugar) where there has been almost a revolution in the trades. Evidently these prices need interpretation by persons conversant with the industries with which they are connected. Silver, on the other hand, is perfectly defined chemically.

Other prices given in the Statistical Abstract are those of wheat, barley and oats, which are obtained by averaging the records of sales in the various corn markets of the country. The prices of minerals, including pig-iron, at the places of their production, can be obtained approximately by dividing their estimated value by the number of tons produced.

5. *Index-numbers.*—When measurable phenomena (such as prices or wages) are influenced (1) by causes *special* to particular instances, (2) by *general* causes presumably acting on all the phenomena, it is important to disentangle the general causes from the special. Thus the price of wheat is influenced by the weather, acreage under the crop, and the harvests in all the wheat-growing countries; the price of coal by the fluctuations in demand: these are causes special to these commodities. The prices of wheat, coal and all commodities are influenced by the relation of the amount of money and its substitutes to the work that has to be done by them: these are general causes. To determine the effect of the general causes, that is, to determine the general change of price, which varies inversely as the purchasing power of gold, it is necessary to eliminate special causes. This is done by

* Even then exceptional years like 1900, when there was a great demand for the coal of South Wales, should be excluded.

averaging together the price changes shown for a number of different commodities, as follows—

As many commodities are taken as possible, for which a perfectly definite price quotation is current, great care being taken to avoid changes of quality; in practice, the number of such commodities is not great, and retail prices must generally be ignored. The average price of a period of years is taken as base, and equated to 100; the prices of other years are then expressed as percentages: *e. g.* from the table above, we should have if we took 1870-79 as base—

	Prices.			Proportionate numbers.		
	Wheat.	Sugar.	Tea.	Wheat.	Sugar.	Tea.
Average 1870–79	11·5	31 9	16·3	100	100	100
Year 1890 . .	7·8	16·3	10·6	68	51	65
„ 1908 . .	8·4	13·0	8·0	73	41	49

The average of the numbers so found for any year is the index number for that year. This is very nearly the method employed by Mr. Sauerbeck to obtain the index-numbers * given in Column 12.

Another method is to follow the process used in the last chapter (pp. 126–8), thus—Imports in 1890 were valued at £356. At the prices of 1902 they would have been worth £324. Prices in 1890 were therefore higher than in 1902 in the ratio $356 : 324 = 110 : 100$. The import index-number for 1890 is therefore 110, when 100 is taken for 1902.

The first of these methods assigns equal importance to each of the commodities chosen, at their average price in the base-period.† To measure the abstract quantity " change of

* To facilitate comparison his index for 1902 is equated to 100, and the rest of the numbers raised in proportion. Since 1914, the numbers have been computed by the *Statist* newspaper on Mr. Sauerbeck's method. Full detail is given each year in the *Statistical Journal.*

† A change of base-period may affect the arithmetical importance of special commodities in the average, as may be seen by taking the three commodities used above and taking 1908 as base; but the difference disappears when many commodities are taken, unless abnormal years are deliberately chosen.

purchasing power " or " appreciation of gold " one commodity is as good as another, and one kind of average is as good as another; it is only necessary to take a sufficient number of commodities to allow the laws of averages free play.

The second method is more objective or concrete; it is used to find the value of a definite group of commodities, and this could be done exactly if the data were sufficient. In such cases the method of index-numbers is only a method of abbreviating computation and overcoming the absence of complete information. It is justified, when it can be shown by the principles of averages that the correct objective result must be approximately reached. Similarly, if we wish to find the change over a period in average wages of several groups combined, we could, if we had complete information, work out the actual average year by year; but, in fact, we can only find the ratio changes for the various groups and have to combine these into a wage index-number by the use of suitable weights. In fact, the method suitable for concrete index-numbers differs from that convenient for abstract index-numbers chiefly because weights must be used for the former (unless it can be shown that they would not affect the result), while they can be very often ignored with the latter.

Consideration will readily show that either of these methods is equivalent to comparing weighted averages of the prices. It was stated with illustrations on pp. 18 and 32 above that errors involved in such a process tended to neutralize each other; supposing there to be one ideal true method, all others may be regarded as differing from it by the introduction of many minor errors. Experience shows abundantly that many different methods of computing price index-numbers yield approximately the same result, when proper care is taken to avoid biassed and preponderant errors.

6. Prior to the War the Board of Trade published a weighted index-number of Wholesale Prices. In 1920 this was discarded, and a new index was constructed in which 150 prices are used, the number of entries for each commodity being proportional to the importance of that commodity in the

national economy as indicated by the Census of Production in 1907. The geometric mean of the 150 price ratios is taken.* This is a convenient and theoretically correct method.

Recent price movements are indicated by the following table, in which 1913 is taken as the base year except for retail prices in the United Kingdom. Some of the numbers already given on p. 140 are repeated with the new base year. The basis of the Retail Food and Cost of Living Index-numbers is explained below (pp. 196–7).

INDEX-NUMBERS OF PRICES, 1913 AND 1920–7

	Wholesale Prices.					Retail Prices.			
	United Kingdom.				United States.	United Kingdom. Ministry of Labour.		United States. Bureau of Labour.	
	Imports.	Exports.	Statist.	Board of Trade.	Bureau of Labour.	Food.	Cost of Living.	Food.	Cost of Living.
1913†	100	100	100	100	100	100	100	100	100
1920	298	359	293	308	226	259	253	204	208
1921	202	268	188	198	147	221	220	153	178
1922	158	199	158	159	149	175	181	141	168
1923	152	190	153	159	154	169	173	146	171
1924	162	191	165	166	150	170	175	146	171
1925	162	184	159	159	158	171	175	157	176
1926	142	174	150	148	151	163	172	160	175
1927	136	164	144	141	146	159	167	155	173

7. Index-numbers may, then, be taken on authority by those who do not desire to follow the extremely interesting analysis on which various methods of obtaining them are

* If we write P_1, P_2 . . . for prices in the year chosen as base, p_1, p_2 . . . for prices in another year, then $r_1 = 100\,\frac{p_1}{P_1}$, $r_2 = 100\,\frac{p_2}{P_2}$ . . . are the price percentages. Write Q_1, Q_2 . . . for the quantities marketed or imported in the base year, and $E_1 = Q_1P_1$, $E_2 = Q_2P_2$ . . . for the values. Then the form of the Statist Index is $\frac{1}{45}(r_1 + r_2 + \ldots)$, of the usual weighted index numbers is $(E_1r_1 + E_2r_2 + \ldots) \div (E_1 + E_2 + \ldots)$, of the Board of Trade number $\sqrt[150]{(r_1 \times r_2 \times \ldots)}$, and of the Import index number $(q_1P_1 \cdot r_1 + q_2P_2 \cdot r_2 + \ldots) \div (q_1P_1 + q_2P_2 + \ldots)$, where q_1, q_2 . . . are the quantities in the particular year.

† July 1914 for United Kingdom Retail Food and Cost of Living.

based, as showing with approximate correctness the general change of prices of the group of commodities to which they relate. Sauerbeck's numbers are typical of wholesale prices of raw materials in the United Kingdom, the export numbers of prices of those commodities (principally manufactured goods) which are exported, the import numbers of the great variety of food, raw materials and other commodities imported. If a diagram is made of these three series, their general resemblance will be marked, and it is an interesting exercise to trace the dates and examine the causes of the differences. It is very important to define the group from which the sample prices are selected.

8. No study of statistical records involving price or value is complete without reference to the general change in the purchasing power of gold thus indicated. The changes may thus be described since 1855: The years 1872–3 were a time of great price inflation, otherwise the general prices fluctuated about the same general level from 1855 to 1874; from 1874 to 1896 prices fell enormously, and but for slight recoveries in 1879–80 and 1887–9, almost continuously, the ratio of prices in 1874 and 1896 being 3 : 2; from 1896 onwards till 1913 a considerable recovery took place, including two sharp inflations and corresponding falls about 1900 and 1904–8. Any discussion of the general change of currency systems since 1913 is outside the scope of this book; the very great fluctuations in all the index-numbers indicate the extent of the inflation till 1920, and the subsequent deflation.

In dealing with the statistics of Eastern and South American countries, it must be remembered that in many cases statistics are given in silver currency, whose value in terms of gold fluctuates with the price of silver shown in the table, p. 140.

CHAPTER V

PRODUCTION

1. The more advanced the stage of manufacture, the further removed from the raw material, the more difficult it is to measure the *quantity* produced by an industry, and the scarcer are the statistics of *value*. We have generally to be content with statistics of the *quantity* of raw material used, and the *value* of that part of the completed goods which are *exported;* for no general continuous record is kept of goods produced for the home market except in the instances given in the next paragraph.

2. The Ministry of Agriculture and Fisheries of England and Wales collects statistics as to the amount of land devoted to various uses, and the estimated yield year by year of the various crops. These are published in the Annual *Agricultural Statistics*, and similar reports are available for Scotland and Ireland, and the results are summarized in considerable detail in the Statistical Abstract. There are no general statistics of the production of wood, fruit, meat or dairy produce, but there are returns as to the numbers of horses, cattle, sheep and pigs. From these latter, combined with expert investigation, estimates have been made of the quantities of meat and dairy produce produced, imported and consumed, and more detail is furnished in recent reports. The weight and value of sea fish landed is estimated year by year.

The statistics as to minerals are good and complete, owing to the fact that mines have long been subject to inspection. The Reports of the Mines Department deal with the quantity and value of the output of coal, copper, lead, tin, zinc and other metals. These figures are summarized in the Statistical

Abstract, which also gives the production of "pig-iron" (the first form in which the metal is obtained from the ore, and the raw material of the iron industries and of a great part of the production of steel) both from British and from foreign ores. The amount of these minerals consumed at home can be obtained by adding the imports to, and subtracting the exports from, the home production, in the case of coal and pig-iron; for other metals special knowledge is needed, since both metal and ore are imported.

The production of pig-iron and steel is estimated as follows—

	Pig-Iron. United Kingdom.	Steel. United Kingdom.
	000's tons.	000's tons.
1891–1895 (average)	7,040	3,030
1896–1900 ,,	8,890	4,590
1901–1905 ,,	8,769	5,125
1906–1910 ,,	9,765	6,164
1911	9,526	6,565
1912	8,751	6,903
1913	10,260	7,670
1920	8,035	9,067
1921	2,616	3,703
1922	4,902	5,881
1923	7,440	8,482
1924	7,307	8,201
1925	6,262	7,385
1926	2,458	3,596
1927	7,294	9,098

The number and net tonnage of ships built in the United Kingdom are quoted in the Statistical Abstract, and distinction is made between ships sold to foreign countries (for mercantile or naval purposes) and those retained under home ownership. (The numbers built for the Royal Navy are not given.) There are great and rapid fluctuations in these totals, and those for single years should never be used in isolation.

3. When dealing with the textile trades we can obtain comparative measurements from the raw material. The import statistics of cotton, jute and silk show, when re-exports are subtracted, the amounts of these fibres brought into the home

market each year; and trade journals show more accurately the amount actually used. For wool the sources are numerous, and the amounts used are shown in the following extract from the annual report of the Bradford Chamber of Commerce.

SUPPLY OF WOOL, ETC. (Million lbs.)

	Foreign and Colonial Wools imported and retained.	Home-grown Wools (total clip less exports).	Wool from imported skins.	Shoddy, etc.	Total quantity of Wool, etc., used.	Estimated value.
						£000,000's.
1880–4 (average)	235	119	20	123	497	20
1885–9 ,,	301	112	25	101	539	18
1890–4 ,,	343	130	32	118	623	20
1895–9 ,,	405	115	33	132	685	21
1900–4 ,,	380	105	29	145	659	20
1905–9 ,,	430	94	36	193	753	27
1910 . .	503	106	37	226	872	32
1911 . .	525	107	34	210	876	32
1912 . .	514	85	33	201	833	30
1913 . .	535	97	35	205	872	33
1920 . .	670	86	33	109	898	64
1921 . .	448	67	28	51	594	32
1922 . .	696	42	21	64	823	42
1923 . .	376	44	19	79	498	27
1924 . .	428	52	22	135	638	51
1925 . .	406	56	18	112	593	49
1926 . .	485	60	22	90	657	—
1927 . .	507	56	26	72	661	—

Of this wool, some is exported after combing, some after spinning, some as cloth or carpets, some as clothes; the remainder is worked up and used at home. Also some foreign yarn is imported. We cannot then say that the total production is proportional to the wool used.

In the case of wool the quantity of output for the home market was in 1926 nearly equal to that for the foreign. In the case of cotton the quantity of exports was about three-quarters of the total quantity. Thus a very rough idea can be obtained of the value of the total product, and the value added in manufacture can be estimated by subtracting that of the raw material.

4. For other industries we have only the incomplete indices afforded by the amount of raw material imported, and by the value and quantity of manufactures exported; since in most cases some of the raw material is produced at home and a great quantity of the manufactures are used at home, and since the proportions of home and foreign production and consumption vary, it is generally prudent not to base any conclusions as to production on such imperfect data. As regards manufactures which contain iron or steel, however, the very considerable increase in the weight and value exported, as shown in the Statistical Abstract, should be noticed.

From the data now described, together with some other unpublished material, a beginning has been made in the construction of an index-number of production,* analogous in some ways to an index-number of prices, by the *London and Cambridge Economic Service* (Memorandum, No. 8, and subsequent Bulletins). In 1927 (*Statistical Journal*, pp. 250 *seq.*) the Board of Trade proposed to establish a more comprehensive computation based on new and more general material and to issue an Index of Industrial Production each quarter. In every such calculation the data must be limited to the simpler kinds of products and the index-number cannot register the continual progress in manufacturing, and especially in engineering, elaboration and improvement.

5. In addition to these statistics of current production, a general Census of Production is taken from time to time. In the United Kingdom the first Census for 1907 and the results were published in 1912 (Cd. 6320); a second Census was

* With the notation of p. 144, the simplest index of production is of the form $(q_1P_1 + q_2P_2 + \ldots) \div (Q_1P_1 + Q_2P_2 + \ldots)$, which equals $\left(E_1\frac{q_1}{Q_1} + E_2\frac{q_2}{Q_2} + \ldots\right) \div (E_1 + E_2 + \ldots)$. The relative values of E_1, $E_2 \ldots$ are estimated from the Census of Production, which form the "weights," while the ratios weighted, $\frac{q_1}{Q_1} \ldots$, are based on whatever quantities are known and are relevant to the corresponding E's.

interrupted by the War; a third Census was taken for 1924, and the preliminary results were published as supplements to the *Board of Trade Journal* in 1927–8. Another Census is expected *circa* 1930. In the United States, Censuses of Production have been taken in conjunction with the Censuses of Population since 1850, and at more frequent dates in recent times,* and the results are published rapidly. The United States give figures for number of establishments, capital employed and totals of wages and salaries, in addition to the items obtained in the United Kingdom. The scope and method are generally similar in the two countries, and both cover all, or very nearly all, manufacturing industry.

The principal statistics obtained are the selling value (or gross output) of the products of each establishment, and the sums paid out for materials, fuel, power, light, etc.; the remainder is termed the "net output" in the United Kingdom and the "value added by manufacture" in the United States. The gross output contains the value of imported materials and of materials accounted for by the establishments producing them, and involves an enormous amount of duplication. The more generally useful and accurate definite total is the "net output." This is the sum from which rents, taxes, royalties, interests, salaries, wages are met (together with depreciation if not otherwise allowed for); profits are the residual when the other items are subtracted.

The nature of this and other information obtained for the Boot and Shoe Industry is shown in the table opposite.

From other sources (see pp. 167–170 below) we learn that average weekly earnings of all employed in the ready-made section of the industry were in 1906 22*s.* 6*d.* for males, 10*s.* 6*d.* for females, 18*s.* 7*d.* for all, and in 1924 54*s.* 11*d.* for males, 30*s.* 10*d.* for females, 45*s.* 4*d.* for all. At the first date, normal weekly hours were 53½, and earnings were reduced by about 5% by short time; at the second date normal weekly hours were 48, and earnings were reduced by about 3% by short time.

* The actual dates of the statistics are 1849, 1859 . . . 1899, 1904, 1909, 1914, 1919, 1921, 1923, 1925.

BOOT AND SHOE INDUSTRY

United Kingdom 1907. Great Britain 1924.

	1907.		1924.	
	Quantity.	Value.	Quantity.	Value.
	1,000 dozen.	£1,000.	1,000 dozen.	£1,000.
Boots, shoes, slippers	8,120	19,874	9,788	47,427
Repairs and miscellaneous.	—	2,873	—	7,957
Gross output		22,747		55,384
Materials, etc.		13,800*		30,351
Net output		9,900†		25,035
Exports	787	2,040	998†	5,024
Imports	186	769	229	1,394
Persons employed :				
Males		89,585		95,898
Females		35,218		51,402
Total		124,800		147,300
Salaried		9,261		16,626
Wage-earners		117,565		130,674
Net output per head		£71		£170

The above comparison should not be used, except for the most general purpose, without reference to the report.

In the table on p. 152, for the United States, the figures subsequent to 1914 are not strictly comparable with those before. The statistics are to be found in the Reports on the *Census of Production* and in the *Statistical Abstract of the United States*. The percentage and per head figures are not given in the Report, but are computed. Earnings per head both in the United Kingdom and in the United States are influenced not only by rates of wages, but also by the changing proportions of numbers in the different industries, and by changes of the relative number of men and women and at different ages. They can only be used within careful

* These can only be given approximately.

† Including 361,000 dozen to Southern Ireland, which would not have been counted as exports in 1907.

	United States.						United Kingdo		
	1904. in $	1909. in $	1914. in $	1919. in $	1923. in $	1925. in $	1907. in £	in $ †	in
Number of establishments (000's)*.	216	268	276	214	196	187	—	—	-
Gross output (000,000) . .	14,790	20,670	24,250	61,740	60,260	62,720	1,765	8,590	3,9
Net output (000,000) . . .	6,290	8,530	9,880	24,750	25,780	26,780	712	3,460	1.7
Number employed (000's) . .	5,988	7,405	8,000	10,419	10,118	9,724	7,090	—	7,8
Wage-earners (000's). . .	5,468	6,615	7,040	8,990	8,768	8,384	6,600	—	7,(
Salaried (000's) . . .	520	790	960	1,429	1,350	1,340	490	—	7
Net output per head . . .	1,050	1,150	1,235	2,380	2,550	2,750	100	487	2
Wages (000,000)	2,610	3,430	4,080	10,450	11,000	10,730	344**	1,670	
Salaries (000,000)	570	940	1,280	2,860	3,000	3,150	60	370	
Wages as % of net output. .	43	40	41	42	43	40	52	-	
Salaries as % of net output .	9·1	11·0	13·0	11·6	11·6	11·8	9	-	
Wages per head per annum . .	477	519	580	1,162	1,255	1,280	52	253	
Salaries per head per annum . .	1,100	1,190	1,320	2,000	2,220	2,350	130	630	
Horse-power of engines (000's) .	13,490	18,670	24,250	29,300	33,060	35,770	10,755	—	21,
Horse-power per wage-earner .	2·5	2·8	3·4	3·3	3·8	4·3	1·6	—	

* Excluding establishments whose net produce was less than $500 in 1904, 1909, and 1914, and less than 1923, 1925. Excluding automobile repairing in 1919.

† Converted at £1 = $4·866 in 1907, £1 = 4·42 in 1924.

** These figures refer to a net output of £659 Mn.; see Text.

limitations, and for particular purposes. Reference should be made to the previous chapter for a general view of the changes in the value of money.

In the statistics relating to the United Kingdom, the figures relating to wages, salaries and percentages are not taken from the Census, but from *The Division of the Product of Industry*, Chapter IV (Clarendon Press, 1928). These figures relate not to the whole Census, but after governmental and other industries which do not show profits are subtracted; the Net Output of the industries subtracted was £53 Mn., and of the part thus treated £659Mn. It is estimated that 9% of the Net Output should be allowed for depreciation, which should be met before the sum is distributed; this exclusion raises the shares accruing to salaries and wages from 52 and 9% to 58 and 10%. If a similar correction ought to be made in the United States statistics the shares of wages and salaries in 1904 would become 45 and 10%. The English figures are converted to dollars at 1£ = $4·866 in 1907 and at £1 = $4·42 in 1924.

The table suggests a number of very interesting comparisons; but they ought to be regarded only as suggestions till the reports have been studied in detail, the definitions of all the terms used established, and the limitations of strict comparability determined.

The analysed statistics for the United Kingdom in 1924 are not available at the time of writing, but a preliminary comparison of the totals can be made.

CENSUSES OF PRODUCTION, UNITED KINGDOM

	1907.	1924.
Output of Industry:	£Mn.	£Mn.
Gross	1,765	3,917
Net	712	1,719
Persons employed:		
Salaried	492,000	768,000
Wage-earners	6,595,000	7,052,000
Net output per head of all employed	£100	£220

The exclusion of Southern Ireland in 1924 makes very little difference, since little manufacture is there carried on.

Wholesale prices, in general, approximately doubled between 1907 and 1924, but it is not safe to apply this ratio to the value of net output, though it is evident that the £220 output per head in 1924 must be very heavily discounted before it is compared with that in 1907.

CHAPTER VI

WAGES

1. STATISTICS of wages are very plentiful; but there are so many different ways of reckoning and paying wages, and such diversity in the methods of stating rates of wages, that these statistics are extremely difficult to handle, and give rise to many misunderstandings.

Wages may be paid by time or by piece. In the former case the rates of wages are so much per hour, per week or other period. The payment does not nominally depend on how much work is done, but there is very often an understanding as to what constitutes an hour's or a day's work, or as to how long a particular job should take. Rates of time wages are very generally agreed on between employers and Trade Unions, and when this is the case there is usually no difficulty in ascertaining them. There is generally also an agreement as to the number of hours which constitute a week's work; the recognized payment for this number of hours is known as the wages for a "normal week," and this payment is the rate generally quoted. It should be observed that in those industries which are carried on at a disadvantage by artificial light the "normal week" is shorter in winter than in summer. Where, as in the building trades, payment is by the hour, there is no certainty that a man will obtain employment for the whole week, and in any case there is loss of time in changing from one job to another in outdoor building work. In this case wage statements generally give the rate per hour and the number of hours which constitute a full week's work season by season. Overtime, that is time outside the scheduled hours that constitute the normal week,

is generally paid for at a higher rate. In some trades overtime is so frequent as to make an important difference in average earnings; in others undertime is common. Besides the week's wage there are in many cases bonuses for regularity or rapidity of work, special rates for special work (*e.g.* harvesting), payments other than money, as when an agricultural labourer has a house at a cheap rate or land to cultivate for himself or perquisites of any kind, or a coal-miner obtains house-coal at a low price. It is thus necessary to have special knowledge of the conditions of employment in each trade before using the bare statements of weekly rates of wages.

2. Piece-rates are, of course, rates of payment for the performance of defined tasks. They very frequently are arranged between employers and employed in the form of elaborate piece-lists (or price-lists, as they are frequently called), which define the exact nature of the task and show innumerable variations of payment corresponding to the various peculiarities of material or machinery by which the work is lightened or made more arduous. The "prices" are usually arranged with a view to the amount of work an ordinary man can do at ordinary pressure in a normal week, so that the week's earnings shall depend rather on the skill or vigour required than on the accident of the special job. Thus printers are paid at a higher rate the smaller the type used. Coal-hewers are paid more per ton when working in narrow seams than where the coal is more easily obtained. Weavers are paid more per yard woven for every additional complexity of the loom. In the large, thoroughly organized industries, especially in the cotton manufacture and in mining, this equalization of earnings is carried to an extraordinary complexity, and the lists are frequently adjusted to suit new conditions as they arise. It is obvious that a piece-list in itself does not give any information as to earnings.

There is in reality no well-defined distinction between payment by piece and payment by time, for time-rates often imply a definite amount of work, and piece-rates are often arranged to produce a definite total of earnings. In fact,

there are many methods of payment which are partly on a time- partly on a piece-basis; for example, in engineering, when time-rates are paid, a definite number of hours is sometimes allotted to a job, these hours are paid for however rapidly it is done, and in addition a bonus paid for rapid work. The distinction between the methods of payment is, however, of statistical importance, for it is much easier to obtain correct accounts of the week's earnings when on a time-basis, for the statements are given in the form wanted, and there is little variation from man to man; while earnings on piece-rates vary greatly according to skill, opportunity and energy, and information has to be obtained by special inquiries as to individual earnings firm by firm; further, in many occupations on a piece-basis the hours of work vary from man to man and from week to week.

3. Changes of piece-rates are, in the larger industries, at any rate, made by a general percentage increase or decrease of the rates paid to many classes of operatives at once. Thus on June 1, 1909, the rates for about 190,000 coal-miners in South Wales and Monmouthshire were decreased "$7\frac{1}{2}\%$, leaving wages $47\frac{1}{2}\%$ above the standard of 1879," that is to say, before the change rates were 55% above those which were arranged in 1879, and have been modified in various details since; the reduction was $7\frac{1}{2}\%$ off these 1879 rates, but $7\frac{1}{2}$ on $147\frac{1}{2}$, *i. e.* only 5% (nearly), on the rates immediately before the change. Where the current rates differ greatly from the standard, it is very important to know on what basis the change is reckoned.

It by no means follows in this case that the ordinary earnings in June 1909 were exactly $47\frac{1}{2}\%$ higher than those in 1879. Rates may remain stationary, while facilities for production improve, or while the normal week is shortened. In the cotton industry, in particular, slight improvements or alterations of machinery are continually being made, which result in greater productiveness by the operatives, with or without additional intensity of work. Sometimes a nominal reduction is exactly counterbalanced, so far as the week's earnings are

concerned, by increased ease of production. In all industries where piece-rates are common it is necessary to make occasional inquiries as to actual earnings under ordinary conditions in a normal week, to check the results shown by the percentage changes of rates. These changes do not always show the exact date nor even the approximate amount of the resulting changes in earnings, for the operatives often try to make the same earnings after a reduction as before by more intense application, or neutralize an increase by more leisurely work, and it is only after an interval that the earnings settle to a new level. Making allowance for these tendencies, it remains true that in general earnings for a full week change nearly at the time of the published dates of change of rates, and roughly in proportion to these when they are considerable.

4. In the case of time-rates, the information available is easier to use. The rates generally quoted are those recognized both by the Trade Unions and by the Masters' Associations (if both are effective bodies), and changes are the result of public negotiations. The Trade Union rate is a minimum below which no member of the union is allowed to accept employment: in recent times, and always in trades where the union was strong, this regulation is actually followed throughout large districts, and even non-union men are unlikely to receive less; but in past times when unions were often weak, the so-called minimum was sometimes a rate which the workmen wished to get recognized, while many were in fact working for less. Special knowledge is, of course, necessary for each trade and district before the actual significance of the rates can be known. It is often supposed that the Trade Union rate is a maximum as well as a minimum; this is not the case in those important industries where there is scope for skill and intelligence; wage-sheets show that payments range several shillings a week above the minimum rate. In fact, it should never be assumed that the Trade Union minimum is the average, or that it bears the same relation to the average over a series of years. As in the case of piece-rates actual inquiries as to earnings must be

made from time to time to correct the impression given by detailed statements of changes.

5. Changes in earnings take place also in many other ways. Where, as in the case of railways or the police, the men are graded and promoted from grade to grade, or receive additional payment in the same grade as their period of service lengthens, a change can be made by an acceleration of promotion or of increase, without any change in the schedule of rates. Where processes of manufacture are changing, it may easily happen that the rates fixed for work at new kinds of machinery result in earnings above or below those made formerly by the operatives who tend them. Such changes are continually taking place in all mechanical industries, and the whole manufacture and the relative numbers at various wage-levels may be revolutionized without a single change of rates taking place. This is only one aspect of a wider process; for in a progressive country some industries are always growing and new industries introduced, while others are stationary or decaying; young persons enter the former trades and find no opening in the latter, and so the population shifts imperceptibly from industry to industry. This tendency results on the whole in an increase in average earnings of the working-class as a group, over and above that shown by changes of earnings in particular industries. Such changes, whether within an industry or in all industries together, can only be measured by occasional complete inquiries as to earnings, combined with estimates of the numbers employed.

6. The official information as to rates of wages is as follows. The Ministry of Labour, and formerly the Labour Department, issues from time to time statements of the time-rates recognized in several industries and in many districts, and also publishes abridgements of price-lists and sliding-scales * in force. The time rates are summarized in the *Abstract of Labour Statistics*.

* Sliding-scales are arrangements (formerly prevalent and still existing in the iron and steel industries) by which recognized rates change by defined amounts in accordance with the rise or fall of the prices realized for the products of an industry. They have in recent years been superseded in important cases by other methods of adjustment.

A report "on changes in rates of wages and hours of labour" used to be published annually, recapitulating and supplementing the details shown monthly in the *Labour Gazette*. More general inquiries (or censuses of wages) were made as to earnings in the years 1886, 1906 and 1924; the results of the former were published in a series of volumes from 1889 to 1893; those of the second are contained in eight reports, of which the last was issued in April 1913; and that for 1924 was summarized in the *Ministry of Labour Gazette*, from June 1926 to July 1927. There have also been special reports on Agricultural Wages. A great part of what is known officially as to general changes of wages is printed and discussed in the three series of *Memoranda relating to British and Foreign Trade and Industrial Conditions*, generally known as the "Fiscal Blue Books" (Cd. 1761, 2337, 4954). The annual publications are full of important information, but they give no data as to the changes indicated in paragraph 5 on the preceding page; in fact, such changes are ostentatiously ignored. The wage census of 1906, used in conjunction with occupation statistics (see pp. 93 *seq.* above), made possible a general view of the result of changes of all kinds since 1886; and that of 1924 yields similar, but less detailed, results for the subsequent period.

7. The following are examples of the information as to changes of wages and hours tabulated by the Ministry of Labour:—

Industry.	Locality.	Date of change in 1928.	Class of workpeople.	Particulars of change.
Glass working	West Riding	Feb. 1	Decorative glassworkers.	Decrease of ½*d.* per hour. Standard rate after change, 1*s.* 7½*d.*
Iron and Steel manufacture	Workington	Feb. 5	Steel millmen	Increase of 1½ per cent. on standard rates, making wages 17½ per cent. above the standard.
Road Transport	Nottingham	Feb. 3	Night loaders	Addition of 4*s.* per week

An estimate is made of the number of persons affected, and the change in the total week's wage bill is computed. Thus, if there were 200 glassworkers affected, and their normal

working week was 48 hours, the effect of the first change named would be $200 \times 48 \times \frac{1}{2}d. = £20$.

Such estimates are summarized every month in the *Gazette*, and an account for the previous year is given each January.

Thus for 1927 we have :—

CHANGES IN WAGE-RATES REPORTED TO THE MINISTRY OF LABOUR, 1927

Industry groups.	Approximate number affected.		Net change in weekly wages.
	Increase.	Decrease.	
Mining and Quarrying . .	18,000	792,000	£—277,150
Brick, Pottery, Chemicals, etc.	1,100	4,000	— 600
Iron and Steel . . .	6,500	135,000	— 25,250
Engineering and Shipbuilding.	185,000	—	+ 18,600
Other Metal	33,000	43,500	— 900
Textile	1,800	232,000	— 23,000
Clothing	170	196,000	— 18,850
Food, Drink, Tobacco . .	630	21,500	— 1,900
Woodworking, etc. . .	800	17,500	— 2,150
Paper, Printing, etc. . .	—	26,000	— 1,950
Building and allied industries.	15,000	57,000	— 2,850
Gas, Water and Electricity Supply	2,600	36,500	— 3,000
Transport	8,000	188,000	— 11,800
Public Administration . .	6,500	66,000	— 5,150
Others	900	38,000	— 2,650
Total	280,000	1,853,000	—358,600

There are perhaps 13,000,000 workpeople whose wage-changes would be recorded in this table, and their weekly wages aggregate about £26 Mn., so that the average reduction over all in the year was between 1 and 2%.

8. It is noticeable how largely coal-mining wages account for the totals. This has been generally the case since the beginning of these records. Wages in the coal industry and in the manufacture of iron and steel change frequently, depending as they do in many cases on the ascertained selling prices of the products. The wages fluctuate more widely than wages in general, and the changes are in no way typical of changes of average wages in the whole sphere of industry.

Unfortunately these changes are the most obvious, and are frequently given too much importance in speeches and writings. The actual rise and fall in these special industries can only be ascertained by observing them over the long period of the ebb and flow of industry. The table below shows the registered changes from the beginning of the series.

NET GAIN OR LOSS TO WEEKLY WAGES YEAR BY YEAR

	Mining and quarrying.		Pig-iron and iron and steel manufacturers.		Textile industries.		Other industries.		Total.	
	Gain. £000's	Loss. £000's	Gain. £000's	Loss. £000's	Gain. £000's	Loss. £000's	Gain. £000's	Loss. £000's	Gain. £000's	Loss. £000's
1893	15	—	—	—	—	1	—	1	13	—
1894	—	47	—	1	—	—	3	—	—	45
1895	—	31	—	—	—	—	3	—	—	28
1896	—	5	2	—	—	—	29	—	26	—
1897	7	—	20	—	—	—	4	—	31	—
1898	58	—	3	—	—	—	20	—	81	—
1899	54	—	14	—	6	—	16	—	90	—
1900	173	—	15	—	6	—	15	—	209	—
1901	—	63	—	19	—	—	5	—	—	77
1902	—	73	1	—	—	—	—	—	—	72
1903	—	33	—	1	—	—	—	4	—	38
1904	—	32	—	3	—	—	—	4	—	39
1905	—	14	2	—	10	—	—	—	—	2
1906	28	—	5	—	13	—	11	—	57	—
1907	176	—	7	—	12	—	6	—	201	—
1908	—	50	—	10	1	—	—	—	—	59
1909	—	56	—	2	—	8	—	3	—	69
1910	6	—	2	—	2	—	5	—	15	—
1911	—	10	1	—	1	—	13	—	35	—
1912	80	—	10	—	15	—	34	—	139	—
1913	105	—	2	—	10	—	61	—	178	—
1914	—	28	—	5	1	—	50	—	18	—
1915	276	—	32	—	56	—	503	—	867	—
1916	238	—	42	—	82	—	523	—	885	—
1917	495	—	95	—	276	—	2,120	—	2,986	—
1918	445	—	71	—	479	—	2,440	—	3,435	—
1919	620	—	150	—	159	—	1,618	—	2,517	—
1920	1,329	—	261	—	650	—	2,553	—	4,793	—
1921	—	2,590	—	477	—	652	—	2,342	—	6,061
1922	—	506	—	241	—	418	—	3,045	—	4,210
1923	122	—	31	—	—	15	—	455	—	317
1924	125	—	17	—	14	—	398	—	554	—
1925	—	67	—	35	—	—	24	—	—	78
1926	64	—	—	4	—	6	—	103	—	49
1927	—	277	—	25	—	23	—	34	—	359
Net aggregates										
1893–1907	213	—	45	—	46	—	103	—	407	—
1908–1913	75	—	3	—	21	—	140	—	239	—
1914–1920	3,375	—	646	—	1,703	—	9,807	—	15,531	—
1921–1927	—	3,129	—	734	—	1,100	—	5,557	—	10,520
1908–1924	601	—	—	21	653	—	4,503	—	5,736	—

A study of the table shows that the net change in the weekly wage bill from 1893 to 1913 was only £646,000, or excluding coal only £358,000. In this period agricultural labourers and railway servants were excluded, and domestic servants are excluded throughout; but the weekly wage bill cannot be put at less than about £10Mn. in 1893 for the purpose of the table, and the figures given presently show an increase of some 19% in average wages in the 21 years, while the table records about 6%. Again, the increase from 1907–1924 in the table cannot be reckoned as allowing more than about 45% in average wages, while the wage-census figures indicate that average earnings had about doubled in this period, and the Ministry of Labour Index-number of wages shows an increase of about 75%.*

The use of these records is of a less general nature; when mining is subtracted, the remainder shows in what years wages were rising and when falling, and to some extent when the movement was rapid and when slow. The more detailed statements relating to separate occupations in separate towns are of the greatest use in making it possible to keep the records of time- and piece-rates up to date.

9. It is convenient that wage-movements before 1914 should be treated separately from those after. With the help of the details of the records now described and similar information from earlier sources, together with a mass of other records of rates of wages and of actual earnings, consecutive accounts for several industries were given in a series of articles in the *Journal of the Royal Statistical Society* by Mr. G. H. Wood and the present author, of which the first appeared in 1895. All results were tentative till checked by the wage census of 1906, but there was sufficient evidence to support the statements of the table on p. 165, as showing the general movements of rates of wages with fair accuracy. It is to be remarked that in the long run wages for work of

* In any case, the total wage bill would grow about 1% per annum from the increase of the population, and this would be additive to any increase shown in the tables above.

any particular grade of skill approximate to each other, so that a sample which includes the most populous industries must be fairly typical of industries all together.

The working up of the data is actually accomplished by means of index-numbers on a basis generally similar to that of price index-numbers, but the details are more complicated and too technical for discussion here. The principle is to take as data the changes recorded, which can be ascertained, rather than the actual earnings, which can be stated in many different ways according to the bias of the informant. Thus in the table opposite the average wage in each industry is taken as 100 in 1880, and the estimated average for other years is given as a percentage of the average in this standard year.*

The column headed "general" shows the course of the average of the wages of all adults employed in all the industries for which the necessary calculations have been made (including the four groups in the following columns), allowing for the shifting from one industry to another and from grade to grade within the industries. The following four columns show similar figures for four important industrial groups. The last column shows the unweighted average (that is, the average of certain rates without reference either to the numerical importance of the different industries, or to the relative growth of some industries) as given in the "Fiscal Blue Books" (Cd. 1761 and 2337), and in the XVIIIth Abstract of Labour Statistics, p. 120.

The general conclusion from any of these columns is that wages were nearly stationary from 1880 to 1887, rose rapidly from 1887 to 1891, were again stationary till 1897, rose rapidly to 1900, fell very slowly till 1905 back to the level of 1899, rose again in 1906–7, and fell in 1908–9; then a considerable rise took place in 1912 and 1913. Wages at the maximum of 1907 were higher than in 1900, and considerably

* 1880 is taken simply for convenience of working. The results shown do not depend at all on what year is taken as standard. The Labour Abstract statistics are transferred proportionally to this base.

higher than at any previous date. Wages at the end of 1913 were probably higher than in 1907.

INDEX-NUMBERS OF AVERAGE RATES OF WAGES

Years.	General.	Textiles.	Agriculture.	Building.	Engineering.	Ministry of Labour. Unweighted Average.
1880	**100**	100	100	100	100	*100*
1881	**100**	104	99	100	103	*102*
1882	**103**	104	97	100	105	*103*
1883	**103**	105	96	100	105	*103*
1884	**103**	105	94	100	104	*102*
1885	**101**	104	93	100	103	*101*
1886	**100**	103	91	100	100	*100*
1887	**101**	104	94	101	101	*100*
1888	**104**	108	96	101	104	*102*
1889	**110**	108	97	103	108	*105*
1890	**114**	111	100	104	111	*109*
1891	**115**	113	100	104	111	*110*
1892	**115**	115	100	105	109	*109*
1893	**115**	115	99	107	108	*109*
1894	**115**	115	99	107	108	*108*
1895	**115**	116	97	108	108	*107*
1896	**115**	116	97	109	111	*109*
1897	**116**	116	99	111	113	*110*
1898	**120**	116	101	112	116	*112*
1899	**123**	120	103	113	119	*115*
1900	**130**	123	109	115	119	*120*
1901	**128**	123	110	115	119	*119*
1902	**126**	123	110	115	118	*118*
1903	**125**	123	110	115	117	*117*
1904	**123**	123	110	115	117	*116*
1905	**123**	127	110	115	117	*117*
1906	**126**	127	110	115	119	*119*
1907	**133**	131	110	115	119	*123*
1908	**130**	131	110	115	117	*122*
1909	**129**	129	110	115	117	*121*
1910	**130**	129	110	115	117	*121*
1911	**131**	129	112	115	119	*122*
1912	**135**	131	114	116	120	*125*
1913	**137**	133	118	119	122	*129*
1914	**138**	133	122	123	122	*130*

The figures subsequent to 1908, except in the last column, are rather roughly interpolated, on the basis of the Labour Department's index-numbers.

On the same basis as that of the first column, the previous maxima and minima for the general average were about—

	Index-number.
1850	68
1855	79
1858	75
1866	90
1868	87
1874	106
1879	99

The numbers in this last table are computed from Mr. G. H. Wood's table, pp. 102–3 of the *Statistical Journal*, 1909.

10. All the statistics of the preceding paragraphs (8 and 9) refer to rates of money wages of persons working full time in a normal week, excluding casual workmen and others not regularly attached to a definite trade. They refer mainly to men, but include the very large numbers of women employed in the textile industries. Two important adjustments must be made before they are applied to measure the economic well-being of the working-class, one for unemployment, the other for the change in the purchasing power of money. The following chapter shows that employment is more regular when wages are rising and *vice versâ*, and that over a long period unemployment in such a group of industries as those considered has neither increased nor diminished perceptibly. The effect of allowing for unemployment would therefore be to increase the fluctuations without affecting the trend of the series shown in the first column of the table on p. 165.

As regards purchasing power in retail commodities, it was stated in Chapter IV above, that the measurement was very difficult; in fact, authorities do not agree as to the movement of prices, especially when rent is included. The following table shows the results of a calculation by the present author.* The prices included are principally those of food. "Real" wages mean wages expressed in terms of commodities, that is, money wages corrected for change in purchasing power. It is very noticeable that periods of rapid increase of wages have

* Adapted from Appendix to *Dictionary of Political Economy*, p. 801.

	Rates of money wages.	Prices.	"Real" wages.†
1852–1870	Rising fast	Rising	Rising considerably in the whole period
1870–1873	Rising very fast	Rising fast	Rising fast
1873–1879	Falling fast	Falling fast	Nearly stationary
1879–1887	Nearly stationary	Falling	Rising
1887–1892	Rising	Rising & falling	Rising
1892–1897	Nearly stationary	Falling	Rising
1897–1900	Rising fast	Rising	Rising
1900–1910	Fluctuating	Rising	Falling
1910–1913	Rising	Rising	? Stationary

been those also of rising prices, which have neutralized to some extent the benefit of the wage-increase; and that periods of stationary wages have been those of falling prices, which have had practically the same effect as an increase of wages with an unchanged price.

11. All the preceding figures apply to averages, not to individual persons. We have extremely few records of the earnings of individuals for periods longer than a week, though information of a difficult and complex nature is accumulating as to the number of weeks' work and the amount of overtime or lost time obtained or obtainable *in a year* in various occupations, and the variation from year to year. On the other hand, we learn from the Wage Census of 1906, the relation of the *weekly* wages and earnings of individuals to the average. The following table shows in abstract form the kind of information obtained. The earnings are those of all persons, whether working full time, overtime, or short time. The men earning less than 15*s*. were in most cases on short time, and were possibly in a few instances earning money also in other places. The boys and girls earning less than 5*s*. in the cotton and woollen industries were generally half-timers. The statistics refer to the returns obtained from all the principal districts for these industries in the United Kingdom. Tables showing the

† For another view see Mr. Wood's article just quoted (*Statistical Journal*, March 1909).

PERCENTAGE OF ALL EMPLOYED, CLASSIFIED BY EARNINGS IN THE LAST WEEK OF SEPTEMBER 1906

	Under 5s.	5s. and under 10s.	10s. and under 15s.	15s. and under 20s.	20s. and under 25s.	25s. and under 30s.	30s. and under 35s.	35s. and under 40s.	40s. and under 45s.	45s. and under 50s.	50s. and over.	Average.
												s. d.
COTTON—												
Men	—	—	2·5	14·0	24·1	19·3	12·8	8·7	8·5	5·6	4·5	29 4
Women	·6	5·0	21·5	34·5	28·3	9·0	1·1	—	—	—	—	18 3
Lads and Boys	11·6	21·7	40·2	21·1	4·2	1·1	·1	—	—	—	—	11 6
Girls	14·3	37·6	33·3	11·1	3·2	·5	—	—	—	—	—	10 0
WOOLLEN AND WORSTED—												
Men	—	—	5·0	14·7	30·3	20·6	17·6	6·1	2·7	·9	2·1	25 11
Women	8	15·5	53·4	22·3	6·8	1·1	·1	—	—	—	—	13 4
Lads and Boys	19·6	43·0	29·7	6·9	·8	—	—	—	—	—	—	8 11
Girls	17·8	54·4	25·9	1·8	·1	—	—	—	—	—	—	8 2
CLOTHING TRADES—												
Men	—	—	6·2	10·4	20·6	24·3	19·3	8·0	5·0	2·0	4·2	28 3
Women	2·0	23·9	43·7	21·4	6·2	1·5	1·3	—	—	—	—	13 0
Lads and Boys	10·2	46·8	27·0	13·2	2·5	·3	—	—	—	—	—	9 7
Girls	39·4	51·7	8·0	·8	·1	—	—	—	—	—	—	5 8

Summary of the Wage Census of 1906 and 1924, United Kingdom

Industrial Groups.	1906.									1924.					
	Annual Wage Bill divided by average number employed in full week.	Average normal weekly hours.	Average earnings in one week.							Average weekly earnings of all workers.			Hours.		
			Those who worked normal hours.		All workers.										
			Males.	Females.	Males.	Females.	Males and Females.			Males.	Females.	Males and Females.	Normal.	Actual.	
	£ s.		s. d.	s. d.	s. d.	s. d.	s. d.			s. d	s. d.	s. d.			
Textiles	43 10	55·3	23 0	13 10	22 11	13 5	17 6			51 6	27 11	37 9	47·9	45·0	
Clothing	37 2	52·7	25 1	11 7	24 2	11 3	14 10			54 10	27 5	35 7	46·7	44·2	
Building, etc.	67 18	53·2	28 4	—	28 4	—	28 4			58 2	—	58 2	45·4	44·7	
Woodworkers	59 15	53·8	24 5	11 3	24 10	10 10	23 10			53 6	26 2	50 6	46·8	45·6	
Metals, Engineering, Shipbuilding	68 10	53·2	28 4	11 0	28 1	10 8	27 5			54 8	25 3	51 7	46·7	46·3	
Food, Drink, Tobacco	48 10	54·1	23 3	9 10	23 5	9 9	19 0			57 2	27 8	46 2	47·5	46·2	
Paper and Printing.	54 0	52·5	26 5	10 2	27 3	9 11	20 10			69 11	27 4	50 6	47·3	46·7	
Earthenware, Chemicals etc.	59 5	53·7	26 0	10 5	25 6	10 2	23 3			56 6	24 8	48 2	47·4	46·2	
Public Utility Services *	67 8	54·7	27 5	14 5	26 5	13 1	26 3			59 5	29 7	58 10	47·6	47·2	
Miscellaneous †	60 0	53·7	25 5	10 11	25 8	10 7	23 7			53 11	26 5	48 10	47·3	46·3	

* Gas, water, electricity and other employees of public authorities.

† The figures in this line are not strictly comparable between 1906 and 1924.

earnings of those who worked the normal week are also given in the Reports. Information was not obtained for mining.

12. The Wage Census of 1924, as that in 1906, depended on voluntary co-operation of employers. For the separate industries the averages are probably sufficiently typical, but since the proportionate number of returns varied from industry to industry, they can only be combined into a general average by the help of general occupation statistics applied in detail. In the summary tables here given, where industries are merged in industrial groups, as in the returns, there is some risk of error from this source. In 1924, less detail was obtained than in 1906 or 1886. Only average earnings and hours in four selected weeks were recorded, and boys and girls were not distinguished from men and women; so that there is no possibility of distributing earners in grades of earnings as on p. 168. Again, in 1906 there were two tabulations, one including only those who worked exactly normal hours, the other including all who were paid in the week; in both cases in one week only. Both sets are shown in the preceding table, and it is seen that overtime and short time nearly balanced. Of course, persons completely unemployed or absent have no place in the averages. In 1924 the actual earnings of all employed in four selected weeks were recorded, with no distinction between normal-time workers and others; but the normal hours in each factory and the hours actually worked were stated, as shown in the last two columns of the table.

Strict comparability between the two accounts could only be obtained by a very careful comparison of details, but the table is sufficient to give a general view of the increase of earnings, most of which took place after 1914, and of the reduction of hours which was effected in 1919 or 1920.

A careful computation based on these and other available data * leads to the conclusion that the change in average earnings between 1911 and 1924 was as follows :—

* *The Division of the Product of Industry*, p. 30, Bowley, Clarendon Press, 1919; and *The National Income in* 1924, Chapter IV, Bowley and Stamp, Clarendon Press, 1927.

AVERAGE WEEKLY EARNINGS IN UNITED KINGDOM OF ALL EMPLOYED

	1911.	1924.
Males	24*s.* 6*d.* = 100	195 = 48*s.*
Females . . .	13*s.* 6*d.* = 100	210 = 26*s.*
All . . .	21*s.* 6*d.* = 100	198 = 42*s.* 6*d.*

The percentages are probably more accurate than the money amounts. The latter are lower than those shown in the table for males owing to the inclusion of agriculture.

13. A different view is obtained if we attend only to changes of rates of wages, not allowing for changes in the relative numbers employed in different industries, or in occupations within the industries, nor for greater facilities for earning by piece-work on improved machinery, by bonuses on production and by other methods, and also by more scientific management. The Ministry of Labour's general statements (*XVIIIth Abstract of Labour Statistics*, pp. 116–20) deal only with nominal changes of time and piece-rates, not with actual earnings, and do not allow for any change in occupations. The two series of index-numbers thus obtained are as follows :—

1906 . . .	91½	1914, July	100
1907 . . .	95	1914, Dec.	101 to 102
1908 . . .	94	1915 „	110 to 115
1909 . . .	93	1916 „	120 to 125
1910 . . .	93½	1917 „	155 to 160
1911 . . .	94	1918 „	195 to 200
1912 . . .	96	1919 „	215 to 220
1913 . . .	99	1920	270 to 280
1914 . . .	100	1921	210 to 215
		1922	170 to 175
		1923	165 to 170
		1924	170 to 175
		1925	175
		1926	175
		1927	170 to 175

The increase here shown between 1911 and 1924 is 83% in rates of wages, instead of the increase of 98% in average earnings.

A great deal of detail is available for studying the movements in separate industries. Owing to the general movement which took place during and after the War towards co-ordination of wages, and agreements on a national scale, such a study is somewhat simpler than before, but the essential difficulties of definition remain.

CHAPTER VII

EMPLOYMENT

1. We are entirely dependent on the Ministry of Labour for statistics of the amount of employment and unemployment. The information falls into three classes, that obtained from Trade Unions, that communicated by employers, and the statistics arising from the operation of the Unemployment Insurance Acts. The Trade Union returns form an unbroken record from the first issue of the *Labour Gazette* by the Labour Department of the Board of Trade in May 1893 till they were unobtrusively dropped after a final appearance in the *Ministry of Labour Gazette* * in January 1927. Reports from employers gradually found a place in the *Labour Gazette*, and from 1905 onwards especially they were given with increased detail, till after the War from motives of economy they were reduced. The statistics of Unemployed Insured Persons began in 1913, following on a short period of returns from the Labour Exchanges, and have become more and more complete as the Acts have been extended.

2. The scope of the Trade Union returns is shown in the table opposite.

The numbers included from coal-mining and textiles are an insignificant proportion of the aggregate in these industries, and contribute little to the total of unemployment thus measured.

* *The Labour Gazette.* The Journal of the Labour Department of the Board of Trade, Vols. I–XII, 1893–1904. The *Board of Trade Labour Gazette*, Vols. XIII–XXIV, 1905–16. The *Labour Gazette*, prepared and edited at the Offices of the Ministry of Labour, 1917 to May, 1922, becoming the *Ministry of Labour Gazette*, June 1922, Vols. XXV *seq.*

"Labour Gazette," October 1909 *

Industries.	Membership of the Unions from whom returns were obtained, September 1909.	Number unemployed at end of September 1909.	Percentage of Membership unemployed.
Building	58,917	6,432	10·9
Coal-mining	139,746	1,669	1·2
Engineering	171,370	18,592	10·8
Shipbuilding	57,280	12,855	22·4
Other metal trades	41,504	2,286	5·5
Textiles	115,821	2,721	2·4
Paper, printing, and bookbinding	59,127	3,820	6·5
Woodworking and furniture	35,165	2,719	7·7
Miscellaneous	16,790	655	3·9
Total	695,720	51,749	7·4

* Supplemented by additional details furnished by the Department.

Among the Trade Unions of the United Kingdom only the minority, who pay allowances to their members when out of work ("unemployed benefit"), keep a record of the members unemployed. Reports were obtained from this minority by the Labour Department of those who are on the unemployed books (whether in benefit or not) of the various branches at the end of each month, together with the membership of these branches. The table just given is compiled directly from these reports. The numbers do not include persons on strike, sick or superannuated, who draw other "benefits" from the unions.

The numbers for the building trades depend only on carpenters and plumbers. The Operative Bricklayers' Society had no unemployed benefit except for travelling. In the winter months carpenters, painters and plumbers have more employment than those in other building operations, and the percentage of unemployment for all the building occupations would be higher in the winter than that shown in the returns. There is also much under-employment, or lost time, in the building trades, where the hourly system

of engagement is prevalent, which is not shown in this table.

On the other hand, the engineering and shipbuilding and, perhaps, the printing trades are adequately represented.

The figures refer almost exclusively to artisans; labourers' unions do not generally pay unemployed benefit.

These returns are, therefore, merely a sample of the facts of unemployment, and there is little reason for taking the resulting percentage as applicable to industry as a whole. It is sometimes supposed that labourers are more frequently unemployed than artisans; but this is not the case when they are attached to industries in which skilled work is prevalent, for the whole group, men and women, boys and girls, skilled and unskilled, co-operate, and the labourers cannot stop unless the work is stopped. Agricultural labourers obtain regular work if attached to a farm, and those who do seasonal work find much the same demand year after year. On the other hand, dock-labour varies considerably.

In many important occupations, however, there is little unemployment.

The percentages shown by these returns can, then, only be used to *measure* unemployment after a troublesome and hazardous estimate; their use is rather to form an *index* of unemployment, which shall reach its maxima and minima at the worst and best times respectively, and fluctuate much or little as the state of the labour market changes is unstable or steady. In the following paragraphs the percentages are used in this sense.

A study of the table on p. 176 shows that unemployment has fluctuated in periods which are nearly decennial, the worst years being 1858, 1868, 1879, 1886, 1893, and 1904, and 1908; in the last two decades the periods are less regular, a long spell of good employment (1886 to 1901) being followed by an abortive crisis in 1904, two fairly good years in 1906, 1907, and bad years in 1908–9.

On the whole, it cannot be said that unemployment as shown by these numbers has either increased or decreased

over a long period; this would be seen better from a diagram than from the averages given, for these depend very much on what period is averaged.

The apparent severity of the worse periods arises from the preponderance of the engineering and shipbuilding trades, some branches of which fluctuate excessively. If these industries are subtracted, the remainder never shows a percentage so high as 6·1. The table on p. 173 above, for October 1909, shows also in that month for engineering and shipbuilding 13·8% unemployed, and for other industries 4·0%. Column D on p. 176 shows the effect of assuming that other industries as a whole are of the same numerical importance as these two.

It is important to notice that the more complete returns obtained in the later years reduce the percentage unemployed by about ·4. In the comparison of statistics subsequent to 1908 with those of an earlier period, great care is necessary. The effect of the newer figures is shown in Column C, from 1898 onwards. The alteration from this adjustment emphasizes the cautions already given as to the difficulty in the use of these percentages in *measuring* unemployment.

3. During the War unemployment became almost negligible, and after the War, from a variety of causes, the scope of the Trade Union Returns altered, and especially after 1923 their comparability became uncertain. The sequence of figures corresponding to those of Columns C and B_1 in the table on p. 176 is

PERCENTAGE UNEMPLOYED

	All industries.	Engineering, etc.		All industries.	Engineering, etc.
1912 .	3·2	3·6	1919	2·4	3·2
1913 .	2·1	2·2	1920	2·4	3·2
1914 .	3·3	3·3	1921	14·8	22·1
1915 .	1·1	0·6	1922	15·2	27·0
1916 .	0·4	0·3	1923	11·3	20·6
1917 .	0·7	0·2	1924	8·1	13·8
1918 .	0·8	0·2	1925	10·5	13·5

From 1921 onwards, pottery trade operatives are excluded, and from July 1924 the building trades also. In the last

Index of Unemployment. Labour Department
Percentage of Trade Unionists Unemployed

	All industries for which Returns are available.				Shipbuilding and Engineering.		Other industries.		Decennial averages.	Notes.
	A.	B.	C.	D.	A_1.	B_1.	A_2.	B_2.	E.	
1851	3·9	—	—	—	3·9	—	—	—	5·2 (9 years)	The numbers in the Columns A are partly based on the expenditures on unemployed benefits (Cd. 2337, p 91).
1852	6·0	—	—	—	6·0	—	—	—		
1853	1·7	—	—	—	1·7	—	—	—		
1854	2·9	—	—	—	2·9	—	—	—		
1855	5·4	—	—	—	5·4	—	—	—		
1856	4·7	—	—	—	4·9	—	1·6	—		
1857	6·0	—	—	—	6·1	—	2·3	—		
1858	11·9	—	—	—	12·2	—	2·5	—		
1859	3·8	—	—	—	3·9	—	1·4	—		
1860	1·9	—	—	—	1·9	—	1·8	—	5·2	The numbers in Columns B are obtained as explained in the previous paragraph.
1861	5·2	—	—	—	5·5	—	1·9	—		
1862	8·4	—	—	—	9·0	—	3·1	—		
1863	6·0	—	—	—	6·7	—	2·7	—		
1864	2 7	—	—	—	3·0	—	·9	—		
1865	2·1	—	—	—	2·4	—	1·2	—		
1866	3 3	—	—	—	3·9	—	1·1	—		
1867	7·4	—	—	—	9·1	—	3·5	—		
1868	7·9	—	—	—	10·0	—	3·5	—		
1869	6·7	—	—	—	8·9	—	3·0	—		
1870	3·9	—	—	—	4·4	—	3·1	—	3·8	The numbers in Column C are the result of further information from certain Trade Unions, and are given in the *Labour Gazette*, January 1909, and January 1914.
1871	1 6	—	—	—	1·3	—	2·0	—		
1872	·9	—	—	—	·9	—	1·0	—		
1873	1·2	—	—	—	1·4	—	·9	—		
1874	1·7	—	—	—	2·3	—	·9	—		
1875	2·4	—	—	—	3·5	—	·9	—		
1876	3 7	—	—	—	5·2	—	1·6	—		
1877	4·7	—	—	—	6·3	—	2·5	—		
1878	6·8	—	—	—	9·0	—	3·5	—		
1879	11·4	—	—	—	15·3	—	6·1	—		
1880	5·5	—	—	—	6·7	—	3·8	—	5·6	The numbers in Column D are the simple averages of those in Columns B_1 and B_2, and are used to reduce the over-preponderance of engineering and shipbuilding in the unadjusted percentages (Cd. 4954, p. 223).
1881	3·5	—	—	—	3·8	—	3·3	—		
1882	2·3	—	—	—	2·3	—	2·4	—		
1883	2·6	—	—	—	2·7	—	2·5	—		
1884	5·1	—	—	—	10·8	—	3·5	—		
1885	9·3	—	—	—	12·9	—	4·2	—		
1886	10·2	—	—	—	13·5	—	5·6	—		
1887	7·6	—	—	—	10·4	—	3·9	—		
1888	4·6	4·9	—	4·1	5·5	6·0	3·4	2·3		
1889	2·1	2·1	—	2 0	2·0	2·3	2·1	1·8		
1890	2·1	2·1	—	2 1	2·4	2·2	1·6	2·0	4·4	The averages in Column E are from Columns A and B or C.
1891	3·2	3·5	—	3·4	4·4	4·1	1·8	2·7		
1892	5·8	6·3	—	6 2	8·2	7·7	2·7	4·7		
1893	—	7·5	—	7 7	—	11·4	—	4·0		
1894	—	6·9	—	7·2	—	11·2	—	3·2		
1895	—	5 8	—	6·0	—	8·2	—	3·8		
1896	—	3·4	—	3 3	—	4·2	—	2·5		
1897	—	3 5	—	3·4	—	4·8	—	2·1		
1898	—	3·0	2·8	2·9	—	4 0	—	1·9		
1899	—	2·4	2·0	2·0	—	2·4	—	1·7		
1900	—	2·9	2·5	2·4	—	2·6	—	2·3	B. about 3·2 C. 4·8	
1901	—	3·8	3·3	3·3	—	3·8	—	2·9		
1902	—	4·4	4·0	4·2	—	5·5	—	2·9		
1903	—	5·1	4·7	5·0	—	6·6	—	3·4		
1904	—	6·5	6·0	6·4	—	8·4	—	4·4		
1905	—	5·4	5·0	5·2	—	6·6	—	3·9		
1906	—	4·1	3·6	3·7	—	4·1	—	3·3		
1907	—	4·2	3·7	3·9	—	4·9	—	3·0		
1908	—	8·1	7 8	8·6	—	12·5	—	4·8		
1909	—	—	7·7	—	—	13·0	—	—		
1910	—	—	4·7	—	—	6 8	—	—		
1911	—	—	3·0	—	—	3·4	—	—		
1912	—	—	3·2	—	—	3·6	—	—		
1913	—	—	2·1	—	—	—	—	—		

years the numbers of coal-miners unemployed affected the movements and totals very considerably, while in the pre-war returns they had very little influence.

4. The annual averages hitherto given are obtained for each year by adding together the percentages shown at the end of each month and dividing by 12, a method which must give very nearly the same result as any more refined calculation possible. If the general average is 4% it means that, so far as the group of occupations which are included is concerned, one person in 25 is on the average unemployed through the year.* This average may be made up by one man in each 25 having no work in the year, or each man losing one week in 25, or any distribution of unemployment between these extremes. We have very little information on this point, but the table on p. 178 (Cd. 2337, p. 101) shows the circumstances for the Amalgamated Society of Engineers.

In this case it is seen that about one quarter of the members bore the whole brunt of unemployment in the bad year 1893.

5. There are good and bad seasons in the year in nearly every occupation. These can be studied, if we average away the peculiarities of particular years as is done in the table on p. 179 :—

Here we see a distinct but small monthly fluctuation when all trades are merged together; the months from March to July are the best, December and January the worst. December or January is the worst month for the separate groups except printing, while the best month varies from March to August.

6. It is interesting and important to study these statistics in times of depression in the light of these seasonal fluctuations.

These numbers were disturbed by strikes in the shipbuilding and engineering trades, which were settled respectively

* Not exactly, for persons losing only a day or two are not registered, and, on the other hand, the names of those who get employment may not be taken off at once.

DETAILS AS TO UNEMPLOYMENT, AMALGAMATED SOCIETY OF ENGINEERS

	1890.	1893.	Mean of the years 1887-1895.
Mean number of members in district, excluding superannuated	6,344	6,934	6,507
Number of separate members unemployed for as much as 3 days some time during the year	1,356	1,832	1,929
Percentage of membership	*21·4*	*26·4*	*29·7*
Average number unemployed at the *same* time during the year	134	706	403
Percentage of membership	*2·1*	*10·2*	*6·1*
Aggregate number of working days lost through want of employment	40,825	215,874	123,166
Number per member	6·4	31·1	18·7
Average number of working days lost by those members who were unemployed for as much as 3 days in the year	30·1	117·8	63·1
Percentage number of members unemployed during the year for—			
Less than 3 days	78·6	73·6	70·4
3 days, and less than 4 weeks	14·1	5·8	12·9
4 weeks, and less than 8 weeks	3·5	2·4	4·6
8 weeks, and less than 12 weeks	1·6	2·1	2·8
Over 12 weeks	2·2	16·2	9·3

in May and September 1908; though persons actually on strike are not included, the influence is always felt beyond the nominal area. From and after December 1908, the figures for "all" are on the new basis named on pp. 175–6 above, and are about ·4 below the height they would otherwise have reached; a slighter effect is produced in the other lines.

It is very noticeable that the two industries of engineering and shipbuilding account for the apparent acuteness of unemployment in 1908 and 1909. The maximum for "others" is not much above the percentage 4·4 reached in 1904 (see p. 176).

In the previous table it is seen that the general percentage for all trades for November is equal to that for October and

PERCENTAGES OF UNEMPLOYMENT, SHOWING THE SEASONAL FLUCTUATIONS

(Compiled principally from the *XIth Abstract of Labour Statistics.*)

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Yearly mean.
All Trade Unions making returns. 1897	3·3	3·0	2·5	2·5	2·3	2 7	2·7	3·5	4·4	4·7	4·8	5·3	3·5
1898	5·0	4·4	3·1	2·9	2·7	2·6	2·6	2·8	2·6	2·5	2·3	2·9	3·0
1899	3·0	2·6	2·5	2·2	2·5	2·3	2·3	2·3	2·4	2·3	2·2	2·5	2·4
1900	2·7	2·9	2·3	2·5	2·4	2·6	2·7	3·0	3·6	3·3	3·2	4·0	2·9
1901	4·0	3·9	3·6	3·8	3·6	3 5	3·4	3·9	3·7	3·7	3 8	4·6	3·8
1902	4·4	4·3	3·7	3·9	4·0	4·2	4·0	4·5	5·0	5·0	4·8	5·5	4·4
1903	5·1	4·8	4·3	4·1	4·0	4·5	4·9	5·5	5·8	5·8	6·0	6·7	5·1
1904	6·6	6·1	6·0	6·0	6·3	5·9	6·1	6·4	6·8	6·8	7·0	7·6	6·5
1905	6·8	6·2	5·6	5·6	5·1	5·2	5·2	5·4	5·3	5·0	4·7	4·9	5·1
1906	4·7	4·4	3·9	3·7	3·6	3·7	3·6	3·8	3·8	4·1	4·5	4·9	4·1
Averages for 10 years 1897-1906.													
All . . .	4·6	4·3	3·75	3·7	**3·65**	3·8	3·75	4·1	4·3	4·35	4·3	**4·9**	4·1
Building (carpenters and plumbers) .	5·4	5·2	4·4	3·6	3·4	3·7	3·5	**3·3**	3·8	4·4	4·8	**5 9**	4·3
Engineering . .	4·5	4·4	4·0	3·8	**3 6**	3·7	3·7	4·0	4·2	4 1	4·5	**5·3**	4·2
Shipbuilding . .	8·0	6·8	6·1	6·3	**6·0**	6·2	6·1	6·8	8·1	9·2	**9·9**	**9·8**	7·1
Printing .	4·8	4·3	**3·6**	4·2	4·5	4·4	3·7	**5 8**	5·1	4·5	2·9	4·2	4·1
Furnishing . .	**7 9**	6·5	3·2	**2 4**	2·5	3·2	4·1	4·1	4·2	4·5	4·9	7·1	4·5
Averages for 7 years 1907-1913. *All*	4·8	4·5	4·4*	4·4	4·2	4·3	4·3	4·5	4 6	4·7	4·4	4·9	4·5

* Eliminating the effect of a coal-mining stoppage in 1912.

less than that for December; but in 1908 there was a fall in November and no change in December, hence October 1908

1907.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Engineering . .	3·2	2·8	2·9	2·8	2·9	3·4	3·5	3·8	4·1	4·5	4·7	6·4
Shipbuilding . .	8·8	7·5	7·1	6·6	6·7	6·4	7·2	9·3	11·7	11·6	12·8	14·2
Others . . .	3·9	3·7	3·3	3·0	3·1	3·4	3·3	3·4	3·8	3·8	4·2	4·8
All	4·2	3·9	3·6	3·3	3·4	3·6	3·7	4·0	4·6	4·7	5·0	6·1
1908.												
Engineering . .	5·8	5·9	7·1	8·6	9·5	10·5	11·1	12·0	12·3	12·7	13·0	14·0
Shipbuilding . .	15·1	20·0	21·5	23·2	26·1	20·6	22·2	25·2	26·6	26·3	25·2	24·7
Others . . .	5·1	4·8	4·9	5·2	5·4	5·6	5·3	5·7	5·4	5·5	5·4	5·5
All	6·2	6·4	6·9	7·5	7·9	8·2	8·2	8·9	9·4	9·5	9·1	9·1*
1909.												
Engineering . .	13·1	12·8	12·6	12·4	12·5	12·1	12·1	11·3	10·8	10 3	9 5	9·8
Shipbuilding . .	23·0	22·5	22·2	23·3	23·4	23·6	23 9	23·3	22·4	21 5	19·3	16·9
Others . . .	5·4	5·0	4·9	4·7	4·3	4·5	4 4	4·5	4·4	4 0	4·0	4·2
All	*8·7*	*8·4*	*8·2*	*8·2*	*7·9*	*7·*	*7·9*	*7·7*	*7·4*	*7·1*	*6·5*	*6·6*

* New basis of computation begins here.

was relatively as well as absolutely the worst month for employment in general. During 1909 there was less than the usual fall in the early part of the year, but August and onwards showed an improvement instead of the usual seasonal increase of unemployment.

In engineering the changes from June 1908 to July 1909 are very nearly proportional to the ordinary seasonal change; no non-seasonal improvement is visible till August 1909. In shipbuilding similarly the first definite sign of improvement is in August 1909.

7. Employers' returns in the *Ministry of Labour Gazette* for March 1928 cover a variety of trades, and use several different types of measurement. For Coal Mines we have the number of wage-earners on colliery books and the average number of days per week in which work is done at the mines; for Iron and for Shale Mines the numbers employed, and similarly, the average number of days. For Blast Furnaces and Tinplate only the number of furnaces, works, or mills in operation are recorded. For Iron and Steel manufacturers we learn the number of workpeople, and the aggregate of individual shifts worked.

In the case of the Cotton, Woollen, Worsted, Carpet, Boot and Shoe, Pottery and Brick Industries, the numbers employed and the total wages paid are given.

Finally, the numbers employed at the London and Liverpool Docks, and the numbers of seamen shipped at the principal ports are recorded.

These returns are not complete for the industries, but are based only on reports from certain numbers of employers, the numbers varying from month to month. The changes are shown for the firms included each month during the month and year that have elapsed.

These statistics of employment and wages have been used with success to measure the change of activity in the trades over long series of years. (*Unemployment in Lancashire*, Chapman and Hallsworth, 1909; *Statistical Journal*, 1912 p. 791; 1927, p. 272; 1928, pp. 158, 182.)

The *Gazette* also contains verbal summaries of the condition of each of the principal industries with these and other available statistics in some detail.

8. The main sources of information about unemployment from at least 1923 onwards are the records of the number of insured persons unemployed. The National Unemployment Insurance scheme covers all industries other than agriculture and domestic service, except a small number who have an equally favourable arrangement of their own. It includes all wage-earners between the ages 16 to 65. Till January 1928 there was no upper limit of age, but since then insured persons on reaching 65 years become entitled to Old Age Pensions, and are no longer included in the statistics. Also salaried and other employed persons who receive less than £250 per annum are included.

The regulations and administration have changed from time to time, and the position of the limit £250 in the scale of salaries varies so as to bring in or exclude numbers of clerks. The exact comparability of the statistics is thus impaired, but they can be used with reasonable caution.

The number of insured persons is counted and classified every summer—the results are generally given in the November *Gazette*—and these figures indicate the general progress or retrogression of an industry. The industries are classified on the same scheme as in the 1921 Census of Population.

The number counted as unemployed is that of the unemployment books lodged at the Labour Exchanges. Books must be lodged for every claim to benefit or when a person ceases to be employed in an insured trade. Persons out of work owing to a trade dispute in which they are directly concerned, and persons who are absent from illness or accident, are not counted among the unemployed.

Summary numbers from the date at which the statistics become of general use are exhibited in the table on p. 182. The actual numbers, rather than the percentages, unemployed are inserted in the main columns, since these give a better view of the relative importances of the constituents in the total.

INSURED PERSONS UNEMPLOYED, GREAT BRITAIN AND NORTH IRELAND

Quarterly Averages.	Males.								Females.	
	Coal.	Iron and Steel..	Engineering.	Shipbuilding.	Building and Construction.	Other Industries.	Total Numbers.	*Percentage of No. insured.*	Number.	*Percentage of No. insured.*
	000's.	000's.	000's.	000's.	000's.	000's.	000's.		000's.	
1923, 3rd	40	54	154	118	108	435	1,047	*12·2*	294	*9·9*
4th	31	50	151	107	118	416	1,016	*11·9*	284	*9·8*
1924, 1st	38	47	134	82	114	401	963	*11·4*	282	*9·4*
2nd	41	46	114	76	82	361	856	*10·1*	241	*8·0*
3rd	94	56	108	75	96	375	943	*11·1*	256	*8·4*
4th	113	61	107	80	106	380	995	*11·7*	272	*8·9*
1925, 1st	125	59	100	83	108	398	1,034	*12·0*	286	*9·2*
2nd	219	62	95	81	77	381	1,058	*12·2*	273	*8·7*
3rd	250	64	96	85	83	394	1,107	*12·7*	290	*9·1*
4th	191	58	95	90	110	371	1,063	*12·2*	241	*7·6*
1926, 1st	119	50	97	88	117	379	1,003	*11·4*	243	*7·6*
2nd	—*	108	121	90	94	513	1,186	*13·5*	335	*10·5*
3rd	—	132	135	96	109	580	1,314	*14·9*	376	*11·8*
4th	—	108	134	100	139	510	1,259	*14·2*	307	*10·2*
1927, 1st	201	41	97	73	134	385	1,082	*12·2*	236	*7·8*
2nd	220	39	75	54	82	320	913	*10·3*	175	*5·8*
3rd	243	41	67	48	92	323	929	*10·4*	194	*6·4*
4th	217	49	69	46	147	335	900	*11·1*	196	*6·4*
1928, 1st †	208	44	67	44	152	350	1,004	*11·7*	201	*6·3*

* Coal Stoppage. Miners unemployed before the stoppage are included in the total columns.
† Excluding persons over 65 years old.

The stress in the particular industries can be judged from the numbers in the third quarter of 1927.

MALES

	Numbers insured, July 1927.	Average Number Unemployed, 3rd quarter, 1927.	*Percentage Unemployed.*
	000's.	000's.	
Coal	1,192	243	*20·0*
Iron and Steel Manufacture	270	41	*15·2*
Engineering (excluding vehicles)	703	67	*9·5*
Shipbuilding	213	48	*22·5*
Building and Construction	1,008	92	*9·2*
Other Manufacturing Industries and Transport	3,703	323	*8·7*
Other occupations	1,810	115	*6·4*
Total	8,899	929	*10·4*

"Other occupations" include Distributive Trades (shop assistants, etc.), Commerce, etc. (clerks), Government, Hotel Service, etc.

In 1923, the whole year in which both measurements were efficient, the Trade Union percentage and the Insurance Unemployment percentage were nearly the same. But, on the whole, it may be expected that the latter would read higher, since it includes a larger proportion of men in casual work. From such measurements as can be made in so short and disturbed a period, it appears that the seasonal variation is similar in the two measurements.

In addition to the monthly statistics there is a weekly return of the numbers registered at the Labour Exchanges as needing work. The relationship between the two totals is explained in the *Ministry of Labour Gazette* (see March 1928, p. 100, in which the effect of the limitation of insurance to persons under 65 years is also traced, p. 96).

The Trade Union Statistics of Unemployment generally ignored persons on part-time, and therefore did not measure the whole stress of want of work. The Ministry of Labour Insurance figures, on the other hand, separate wholly unemployed from those who are entered under the heading:

Temporary Stoppages. These figures "include those persons recorded as unemployed on the date of the return who were either on short time or who were otherwise stood off or suspended on the definite understanding that they were to return to their former employment within a period of six weeks from the date of suspension." Thus where part-time was organized by the closing of works one week in three, or two days in the week, etc., a due proportion of those affected are included. But if part-time was the loss of some hours in one day, and consequently those affected were not registered as unemployed, there would be no proportional entry.

Important industries where organised part-time was considerable in February 1928 are shown separately in the adjoining table.

INSURED PERSONS UNEMPLOYED IN GREAT BRITAIN AND NORTH IRELAND, FEBRUARY 1928

Industry.	Number Insured.	Wholly Unemployed.	Temporary Stoppages.
		Males.	
	000's.	000's.	000's.
Coal	1,158	144	71
Iron, Steel, and Tinplate	243	23	29
Cotton	202	9	8
All Industries	8,576	836	190
		Females.	
Cotton	360	11	20
Wool	144	3	7
Tailoring	131	4	5
All Industries	3,208	131	70

Three important investigations by sample have been made of the age and other facts relating to persons claiming unemployment benefit, and of the number of contributions paid, number of weeks' benefit drawn, etc. The first is summarized in the *XVIIth Abstract of Labour Statistics*, pp. 74 *seq.*; the second and third were published in 1927 as a "Report on an investigation into the employment and insurance history of a sample of persons insured against unemployment in Great Britain," and under a somewhat similar title in 1928.

9. There are no comprehensive statistics of unemployment in the United States, and consequently the number out of work tends to be greatly exaggerated in times of depression. The best comparative account relates to unemployment among a group that numbered only 250,000 workers in 1920.

ORGANISED WAGE-EARNERS IN MASSACHUSETTS
(Bureau of Labour Statistics, Bulletin 310, 1922).
PERCENTAGE UNEMPLOYED OWING TO LACK OF WORK OR MATERIAL.

	March.	June.	September.	December.
1908 . . .	16·2	12·5	8·7	11·0
1909 . . .	9·5	4·6	3·4	4·9
1910 . . .	5·3	5·4	4·0	7·3
1911 . . .	7·5	4·2	3·7	6·0
1912 . . .	5·1	3·4	3·0	6·4
1913 . . .	7·3	4·3	4·3	7·3
1914 . . .	9·2	6·9	8·5	14·9
1915 . . .	12·8	7·6	3·6	4·0
1916 . . .	3·9	1·3	1·9	2·7
1917 . . .	3·7	3·5	2·7	3·5
1918 . . .	3·0	1·0	1·1	5·3
1919 . . .	11·2	2·7	2·5	3·8
1920 . . .	3·4	14·2	16·1	28·7
1921 . . .	21·8	19·9	18·8	23·4

The percentages in March and December are seen to be generally higher than those in June and September.

CHAPTER VIII

OTHER STATISTICS RELATING TO THE WORKING CLASSES

1. BESIDES the statistics of occupation, production, wages and employment already dealt with, there are several other statements relating to the working-class, most of which are summarized in the *Abstracts of Labour Statistics*. We will omit the statistics of profit-sharing, of industrial accidents, and of diseases of occupations, and deal briefly with the tables relating to Trade Disputes, Trade Unions, Friendly Societies, Co-operation, and Cost of Living.

The statistics relating to strikes and lock-outs are obtained directly from the employers and Trade Unions concerned during and at the end of the dispute. Apart from information as to the wages and normal hours of labour recognized before and after, and as to changes of any kinds made in the conditions of employment or working arrangements, the statistics collected relate to the causes and to the results of the disputes and to the methods by which they were terminated, to the number of persons directly or indirectly affected, and to the number of working-days lost.

By the number of persons *directly* affected is meant those who are actually on strike or locked-out; in the number *indirectly* affected are included "other workpeople employed at the establishments where the dispute occurred, and thrown out of work by the dispute." Clearly this latter category is arbitrary; if carpenters were the permanent servants of a firm whose works were closed they would be classed as "indirectly" affected, whereas if they were hired through a contractor as required they would be equally affected by

the loss of work, but would not be included. In fact the effect of a strike cannot be measured; members of all the industries, at home or abroad, who furnish material for the manufactures which are stopped or use their finished products, and at a later stage the great multitude of people who in general provide the strikers with commodities which they can no longer afford when their wages stop, are to a greater or less extent thrown out of employment; the effects of a strike spread through industry like ripples over a pool when a stone is dropped into it.

The number directly affected is rendered indefinite by the difficulty in distinguishing them on any definition from those indirectly affected. If weavers are on strike, the sizers and dressers may cease work either because they sympathize with the weavers' grievances, or because their work is useless when the looms are stopped, or because the employer locks out all hands. The effect is much the same, but in the first case they are "directly," in the others "indirectly" affected.

This difficulty of definition cannot be got over, and therefore the statistics, and others based on them, can only be used as indications of the effect of disputes, and, with due caution, for comparing one year with another.

The number of days lost through a dispute is computed from the number of workpeople stopping work and the duration of the stoppage. This is a little fictitious, for there is no certainty that this number would have obtained work throughout if there had been no stoppage, and it is probable that either there will be extra work to do after the dispute, or that more work has been done in other places during the dispute, or that trade has been permanently displaced. These criticisms have yet more force when the loss of wages is computed, as is sometimes done unofficially in this country and officially in others.

In fact, the circumstances of strikes cannot be made the subject of exact statistics; we can only note in general terms whether they are becoming more or less acute as the

years go on. The following table shows the principal statistics for the United Kingdom for 1893–1927.

	No. of disputes.	No. of work-people directly affected.	No. of work-people indirectly affected.	Aggregate duration in working-days, *i.e.* total number of days lost by persons directly or indirectly affected.*	Percentage number of disputes settled.		
					In favour of work-people.	In favour of employers.	Compromised or indefinite.
		000's	000's	0,000's			
1893	615	594	40	30,47	40	34	26
1894	929	257	68	9,53	35	36	29
1895	745	207	56	5,72	35	37	28
1896	926	148	50	3,75	41	33	26
1897	864	167	63	10,35	38	36	26
1898	711	201	53	15,29	33	32	35
1899	719	138	42	2,52	32	35	33
1900	648	135	53	3,15	31	34	35
1901	642	111	68	4,14	25	44	31
1902	442	117	140	3,48	24	47	29
1903	387	94	23	2,34	23	48	29
1904	355	56	31	1,48	17	51	32
1905	358	68	26	2,47	20	46	34
1906	486	158	60	3,03	31	37	32
1907	601	101	47	2,16	32	41	27
1908	399	224	72	10,83	20	44	36
1909	436	170	131	2,77	18	46	36
1910	531	385	130	9,89	25	37	38
1911	903	831	131	10,32	25	32	43
1912	857	1,233	230	40,91	27½	30½	42
1913	1,497	516	173	11,63	29	25	46
			Excluding Southern Ireland.				
1912	834	1,232	230	40,89	27·6	29·7	42·7
1913	1,459	497	167	9,81	29·1	25·4	45·5
1919	1,352	2,401	190	34,97	25·5	22·6	51·9
1920	1,607	1,779	153	26,57	24·3	31·5	44·2
1921	763	1,770	31	85,87	19·9	41·3	38·8
1922	576	512	40	19,85	19·3	38·5	42·2
1923	628	343	62	10,67	29·8	29·1	41·1
1924	710	558	55	8,42	23·0	33·1	43·9
1925	603	401	40	7,95	26·1	31·1	42·8
1926	323	2,724	10	162,23	20·8	38·4	40·8
1927	303	89	18	1,18	19·8	38·3	41·9

* This includes in each year days lost by persons through disputes which began in a previous year.

The high numbers of working days lost were mainly due in 1893 to the strike of coal-miners in the Federated Districts, in 1894 to the Scottish coal-miners' dispute, in 1897–8 to the engineers' dispute, in 1908 to the shipbuilders' and South Wales coal-miners' and cotton-operatives' disputes, in 1911 to the railway strike, in 1912, 1920 and 1921 to the coal strikes, and in 1926 to the general strike and coal-dispute.

The series in this table do not show any very definite trend, nor any clear connection with the periods of good or bad trade or of rising or falling wages.

2. The statistics relating to Trade Unions have been for many years good and complete. In general, very careful accounts are kept in detail of membership, receipts and expenditure by the officials of the various unions, and are published periodically for the information of their members. The more interesting details are summarized for principal Trade Unions in the Abstract of Labour Statistics, and are to be found in the Annual Reports of the Chief Registrar of Friendly Societies.

	Number and membership of all Unions from which information is received.		100 Principal Unions.											
						Expenditure on various "benefits."								
	Number.	Membership.	Membership.	Funds at end of year.	Income.	Unemployed.	Dispute.	Sick & Accident.	Superannuation.	Funeral.	Miscellaneous.	Working and other expenses.	Total expenditure.
		000's	000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's
1899	1310	1,861	1,164	3,226	1,835	187	120	287	174	90	69	327	1,253
1900	1302	1,972	1,210	3,733	1,950	263	140	308	184	95	92	362	1,443
1901	1297	1,979	1,219	4,139	2,050	328	210	326	198	95	101	387	1,644
1902	1267	1,966	1,217	4,426	2,094	433	221	341	217	95	95	405	1,807
1903	1255	1,942	1,206	4,612	2,109	517	176	364	238	93	96	439	1 923
1904	1229	1,911	1,203	4,680	2,124	660	118	388	265	96	102	426	2,056
1905	1228	1,934	1,220	4.830	2,228	529	214	403	286	93	116	432	2,078
1906	1250	2,129	1,307	5,222	2,364	429	154	415	306	99	105	465	1.972
1907	1243	2,425	1,471	5,668	2,518	469	138	434	328	105	113	486	2,072
1908	1218	2,389	1,451	5,201	2,767	1,026	606	467	355	107	138	534	3,234
1909	1199	2,369	1,437	5,079	2,585	952	156	440	376	107	145	529	2,707
1910	1195	2,446	1,472	5,153	2,716	702	352	419	403	104	138	524	2,642
1911	1204	3,019	1,821	5,595	2,952	457	318	436	412	113	197	578	2 510
1912	1149	3,288	2,000	5,002	3,280	593	1,375	440	425	119	163	703	3,823
1913	1135	3,987	Particulars not yet available.										

The relatively small amounts spent on Dispute Benefit as contrasted with the amounts on unemployment, superannuation, sickness and accidents are very noticeable.

At the end of 1911 the funds in possession of these unions was £5,595,000, only about 2¼ times the annual expenditure. Of this total £1,699,000 was held by 16 unions connected with mining and quarrying, and £1,420,000 by 15 unions in the metal, engineering, and shipbuilding trades. Similar statistics for a larger number of Trade Unions can be given from 1910 onwards.

TRADE UNIONS REGISTERED BY THE CHIEF REGISTRAR OF FRIENDLY SOCIETIES, GREAT BRITAIN

Year.	Number of Unions.	Membership.	Funds at end of year.	Income from members.	Expenditure on various benefits.					Management and other expenses.	Total expenditure.*	Membership of all Trade Unions in Great Britain and Northern Ireland.
					Unemployed.*	Dispute.	Sick and Accident.	Funeral.	Miscellaneous.			
		000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	£000's	000's
1910	537	1,981	5,871	2,746	677	530	486	120	609	682	3,104	2,565
1911	529	2,321	6,294	3,220	478	603	506	128	741	737	3,193	3,139
1912	521	2,547	5,589	3,458	630	1,655	512	137	689	912	4,535	3,416
1913	535	3,205	6,471	4,091	507	446	704	149	733	1,120	3,659	4,135
1920	568	6,929	15,866	11,196	1,405	3,219	747	295	2,580	4,275	12,521	8,337
1921	535	5,454	10,814	11,314	7,317	3,427	978	321	1,727	4,401	18,171	6,621
1922	506	4,506	9,861	8,865	2,911	1,428	907	316	1,563	3,753	10,878	5,616
1923	491	4,369	10,752	7,985	1,084	721	780	284	1,550	3,225	7,644	5,413
1924	484	4,458	11,434	8,236	1,069	1,188	819	307	1,868	3,232	8,481	5,534
1925	488	4,448	12,556	7,986	1,406	313	789	318	1,519	3,196	7,541	5,497

* Subtracting sums received from the Ministry of Labour for Unemployment Insurance and Administration expenses.

3. The Friendly Societies have in the aggregate very much larger funds and a much greater membership than Trade Unions. The methods, objects and importance of the very large number of societies registered vary so much that the gross totals show very little. Here attention is confined to non-collecting Societies.*

* Collecting Societies include the Assurance Societies, which collect sums from a very great number of the working-class, principally for Funeral "Benefit."

FRIENDLY SOCIETIES PROVIDING SICKNESS OR DEATH BENEFITS
(Excluding medical and collecting societies)

	1910.	1920.	1922.
Number of societies and branches	26,516	23,286	22,564
Membership (000's) . .	6,307	7,216	7,479
Accumulated funds (£Mn.) .	49·7	67·5	75·5
Sickness pay (£Mn.) . . .	3·66	3·13	4·05
Sums at death „ . . .	0·89	1·03	1·18
Other benefits „ . . .	1·16	1·92	2·44
Total benefits „ . . .	5·71	6·08	7·67

4. The Registry of Friendly Societies also receives information as to Building Societies and as to Co-operative Societies, which (together with other information specially collected) is summarized in the *Abstracts of Labour Statistics*. The former are not confined to the working-class, and the statistics are not easy to interpret. The latter hold a very important part of the aggregate of working-class savings, and no small proportion of working-class expenditure is accounted for in their statistics of sales. The following tables contain some summary statistics of these societies :—

ALL CO-OPERATIVE SOCIETIES IN THE UNITED KINGDOM
for which information is received

	1898.	1903.	1908.	1912.
	000's	000's	000's	000's
Number of Members . .	1,596	2,089	2,526	2,898
Capital, Share . . .	£19,280	£26,601	£33,082	£38,403
Capital, Loan . . .	£4,983	£7,994	£10,772	£13,742
Amount of Sales . . .	£70,347	£99,122	£128,752	£151,015
Sales by Retail distribution societies	£42,582	£57,513	£69,786	£83,607
Sales by the English Wholesale society's distributive departments . . .	£12,575	£19,333	£24,903	£31,372
Sales by the Scottish Wholesale society's distributive departments . . .	£4,692	£6,395	£7,531	£8,964

INDUSTRIAL CO-OPERATIVE SOCIETIES IN GREAT BRITAIN

	1912.	1924.
	000's	000's
Retail Societies :		
Members	2,749	4,663
Sales	£80,311	£176,562
Wholesale Societies :		
Sales	£38,126	£90,273
Productive Trading Societies :		
Sales	£3,988	£6,235

For comparison with these figures it may be added that the total paid in wages in the United Kingdom was estimated roughly at about £800 Mn. per annum in 1911, and £1,600 Mn. in 1924 (excluding S. Ireland). Of course sales are not exclusively to the working-class.

The statistics of the last three paragraphs suggest a very interesting investigation, beyond the scope of the present work, into the aggregate savings of the working-class.

5. A great deal of attention has been given from time to time in various countries to working-class "budgets," which show the cost and amount of the various commodities on which wages are spent. The information is always collected first-hand from the workman or his wife, and it is not easy to secure accurate accounts either of income or expenditure, since to include clothes and occasional earnings these accounts should be spread over a long period, an undertaking that requires intelligence, time and attention to minutiæ on the part of the informant. Often, in fact, the budgets do not exactly balance; expenditure on drink and luxuries tends to be underestimated, and in the end the returns apply only to specially thrifty households. So far as the items contained in the following table are concerned these objections do not apply. The tables are taken from the Reports on the Cost of Living of the Working Class in the United Kingdom, France and Germany (Cd. 3864, 4032 and 4512); these volumes contain a great wealth of detailed information about wages, prices, rents and conditions in a large number of towns

in the countries named. The relative levels of wages, rents and prices are compared by means of a system of index-numbers which presents many statistical difficulties, and the results based on them should be criticized closely in relation to the data. As soon as we attempt to compare the well-being of two groups of people we find that statistics of incomes, prices and methods of expenditure only take us part of the way; habits, desires, thriftiness and skill in domestic economy vary greatly from class to class and from nation to nation, and cannot be reduced to statistical measurement; but the data in these volumes make possible vivid descriptions of the economic life of typical working-class families throughout the towns of the three nations.

Similar Reports (Cd. 5065 and 5609) have been published relating to Belgium and U.S.A.

The figures given in the tables on pp. 194 and 195, however, present few statistical difficulties.

It must be remembered that those tables do not show the relative levels of wages in the three countries, but only the distribution of expenditure of wages of given amounts. The general agreement between the three tables is remarkable; in each country the proportion spent on food falls by about one-sixth part as incomes increase from the lowest to the highest; the *proportion* of the food expenditure devoted to meat is nearly the same for all incomes, while the *actual* expenditure increases. A difference is noticeable as to the consumption of bread and flour; in the United Kingdom it is from 28·5 to 30 lbs. except for the large families shown in the last column; in France and in Germany, on the other hand, the lower the income the less bread consumed per head. Among many interesting details we may notice the large amount of sugar consumed in the United Kingdom.

AVERAGE WEEKLY COST AND QUANTITY OF FOOD CONSUMED BY URBAN WORKMEN'S FAMILIES, UNITED KINGDOM, 1904

Limits of weekly income	Under 25s.	25s. and under 30s.	30s. and under 35s.	35s. and under 40s.	40s. and above.
Average weekly family income	21s. 4¼d.	26s. 11¾d.	31s. 11¼d.	36s. 6¼d.	52s. 0½d.
Average number of children living at home	3·1	3·3	3·2	3·4	4·4
COST.					
Bread and flour	s. d. 3 0½	s. d. 3 3¾	s. d. 3 3½	s. d. 3 4½	s. d. 4 3¾
Meat (bought by weight)	2 8	3 4¾	4 3½	4 5½	5 10½
Other meat and fish	0 7½	0 8¾	0 10	1 0	1 4
Bacon	0 6¾	0 9	0 10½	0 11½	1 3¾
Eggs	0 5¾	0 8½	0 11	1 0	1 4¾
Fresh milk	0 8	0 11¾	1 3½	1 4½	1 7½
Cheese	0 4¾	0 5¾	0 6	0 6	0 8
Butter	1 2	1 7	1 10½	2 0	2 0½
Potatoes	0 8¾	0 9¾	0 10½	0 10½	1 1¾
Other vegetables and fruit	0 4¾	0 7	0 10	0 11¾	1 3½
Rice, tapioca and oatmeal	0 4½	0 5	0 6	0 5¾	0 7
Sugar	0 8	0 10	0 10¾	0 11½	1 3
Tea	0 9½	0 11½	1 0¾	1 1½	1 5
Coffee and cocoa	0 2	0 3½	0 3½	0 4½	0 5½
Jam, etc.	0 4½	0 5½	0 6	0 6½	0 8¾
Other items	1 4	1 7¾	2 0	2 5	3 2½
Total expenditure on food	14 4¾	17 10½	20 9½	22 8½	29 8
Expenditure on bread and flour, as % of food cost	21	19	16	15	15
Expenditure on fish, meat and bacon, as % of food cost	27	27	29	29	28
Expenditure on all food, as % of income	67	66	65	61	57
QUANTITIES.					
Bread and flour	lbs. 28·4	lbs. 30·0	lbs. 29·4	lbs. 30·0	lbs. 37·8
Meat (bought by weight)	4·4	5·3	6·3	6·4	8·2
Bacon	·9	1·1	1·2	1·4	1·8
Cheese	·7	·7	·8	·8	1·0
Butter	1·1	1·5	1·7	1·9	2·8
Potatoes	14·0	15·8	16·1	15·9	19·9
Tea	·48	·55	·57	·59	·72
Sugar	3·9	4·6	4·8	5·2	6·7
Fresh milk	pints. 5·5	pints. 7·7	pints. 9·8	pints. 10·3	pints. 12·6

FRENCH TOWNS. 1907–8

Limits of weekly income	Under 20s.	20s. and under 25s.	25s. and under 30s.	30s. and under 35s.	35s. and under 40s.	40s. and above.
Average weekly family income	17s. 9½d.	22s. 11d.	27s. 7¾d.	32s. 4½d.	37s. 8¾d.	52s. 11d.
Average number of children living at home	1·6	1·8	1·8	1·9	2·1	2·9
COST.						
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
Bread	2 9	2 11¾	3 1½	3 4½	3 8	4 8½
Potatoes	0 7¾	0 7½	0 7½	0 8	0 9	0 11½
Meat, bacon, etc.	3 2	4 2½	5 0¾	6 2½	6 11	9 4½
Sugar	0 4½	0 5½	0 5½	0 6	0 6½	0 7½
Other items	4 2¾	5 8	6 11½	8 0½	9 1	12 8½
Total expenditure on food	11 1¾	13 11	16 2½	18 8¾	20 11½	27 11½
Expenditure on bread, as % of food cost	24	21	19	18	18	17
Expenditure on meat, etc., as % of food cost	28	30	31	33	33	33
Expenditure on all food, as % of income	68	61	59	58	56	58
QUANTITIES.						
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Bread	22·9	24·1	24·6	26·2	27·6	35·9
Potatoes	14·7	12·3	13·9	14·6	15·8	20·5
Meat	4·2	5·5	6·5	7·8	8·6	11·5
Sugar	1·2	1·5	1·5	1·7	1·8	2·2

GERMAN TOWNS. 1906–7

Limits of weekly income	Under 20s.	20s. and under 25s.	25s. and under 30s.	30s and under 35s.	35s. and under 40s.	40s. and above.
Average weekly family income	17s. 7¾d.	22s. 8¼d.	27s. 1d.	31s. 10¼d.	36s. 8d.	48s. 8¼d.
Average number of children living at home	2·4	2·3	2·5	2·5	2·8	3·8
COST.						
	s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
Bread and flour	2 5½	2 7¾	2 10½	3 0½	3 5	4 6½
Potatoes	0 9¾	0 10	0 10½	0 10½	0 11¾	1 2½
Meat, bacon, etc.	3 11	4 4½	5 0½	6 0	6 8¾	8 11¾
Sugar	0 4¼	0 4½	0 4¾	0 5	0 5¼	0 6½
Other items	4 7	6 4¾	7 8½	8 6½	9 7	12 1½
Total expenditure on food	12 1½	14 7½	16 10½	18 10½	21 1¾	27 4½
Expenditure on bread and flour, as % of food cost	20	18	17	16	16	16
Expenditure on meat, etc., as % of food cost	32	30	30	32	32	33
Expenditure on all food, as % of income	69	64	62	59	58	56
QUANTITIES.						
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Bread and flour	22·0	23·5	25·0	26·1	29 8	38·2
Potatoes	28·1	25·6	24·0	23·8	24·6	33·5
Meat, bacon, etc.	4·9	5·6	6·2	7·4	8·2	10·8
Sugar	1·8	1·8	2·0	2·0	2·1	2·7

6. From the beginning of the War much more detail has been obtained for Cost of Living Statistics.

In the United Kingdom the Budget, which is the average of those shown on p. 194, was slightly revised, estimates were made of the relative importance of food, rent, clothing, etc. in working-class expenditure, and statistics of prices have been collected monthly, and published in the *Ministry of Labour Gazette.*

In the United States working-class budgets have been collected on a more elaborate scale more than once.* The basis of the existing computation is a collection of 12,096 families made in 1918.

Owing to the difference in dates to which the budgets relate and to the complexity and change of the United States budgets, it is not easy to make a comparison of the cost of living in the two countries, nor of the relative importance of different commodities, but a general view can be obtained.

FOOD BUDGETS IN UNITED KINGDOM AND THE UNITED STATES
Relative importance of expenditure on different foods

	United Kingdom.		United States 1918.	
	1913.	1928.	All.	Excluding vegetables and fruit.
Meat, lard, fish	300	279	258	276
Eggs	56	55	82	88
Milk, butter, margarine, cheese	257	265	256	274
Bread, flour, cereals	210	214	210	225
Sugar	57	64	34	37
Tea, coffee	66	68	41	44
Potatoes	54	55	53	56
Vegetables and fruit	0	0	66	—
	1,000	1,000	1,000	1,000

* See Bulletins of the Bureau of Labour Statistics, Nos. 357 and 366. *The Statistical Abstract of the United States for* 1926, pp. 321–5, and *The Cost of Living in the United States*, National Industrial Conference Board, New York, 1926.

The 1928 figures for the United Kingdom are obtained by assuming that the same quantities of the various foods are bought as in 1913, which is known to be approximately true, and applying the price changes stated in the *Gazette* to each item separately. Thus the price of sugar has risen more than the price of meat, and, therefore, on this assumption the expenditure on sugar has become a larger proportion of that on meat at the later date.

In the United Kingdom budget no vegetables or fruit except potatoes are included. Otherwise the budgets cover similar ranges of food, but that of the United States is more detailed.

The budget on which the Cost of Living computation is based include also rent,* clothing, fuel and miscellaneous items. In the United Kingdom only a few entries for cleaning materials and utensils are included in the last category.

HOUSEHOLD BUDGETS IN THE UNITED KINGDOM AND THE UNITED STATES

Relative expenditure in different categories

	United Kingdom.		United States.	
	1913.	1928.	1918.	
			All.	Reducing miscellaneou items.
Food	60	57	38	50
Rent	16	15	13½	17½
Clothing	12	16	16½	21½
Fuel and light	8	8	5½	7
Miscellaneous	4	4	21½	4
Furniture, etc.	—	—	5	
	100	100	100	100

The last column is obtained by putting "miscellaneous" at 4 per 100, and redistributing the other entries in proportion. The percentages for the United Kingdom in 1928 are obtained as in the previous table.

* The United States entry is housing, and includes items besides rent. In United Kingdom rates are included in rents

The index-numbers for June 1925 are obtained as follows:—

CHANGE IN THE COST OF LIVING

	United Kingdom.				United States.			
	Relative importance.		Price level in June 1925 as percentage of July 1914.	Product.	Relative importance.		Price level in June 1925 as percentage of 1913.	Product.
Food	60	×	167 =	10,020	38	×	155 =	5,890
Rent	16		147	2,352	13½		167	2,254
Clothing	12		230	2,760	16½		171	2,822
Fuel	8		180	1,440	5½		177	974
Miscellaneous	4		180	720	21½		202	4,343
Furniture	—		—	—	5		214	1,070
Total	100			17,292	100			17,353

The index-numbers for June 1925 are then 17,292 and 17,353 divided by 100, viz. 173 and 173·5; that is, the Cost of Living was computed to have risen 73% and 73½% in the two countries. If, however, we reduce the importance of the miscellaneous items in the United States to 4% as before, and also adjust to 1914 (when prices were about 2% higher than in 1913) on the base, we obtain 165, *i. e.* an increase of 65% in the United States.

7. We have not dealt with Statistics of *Pauperism*, because they are likely to be very misleading, from incomplete and faulty definition, unless handled with special care and knowledge.

Education statistics are plentiful and accessible in the reports of the Board of Education.

Statistics relating to *Old Age Pensions* are summarized from Reports of the Commissioners of Customs and Excise in the *Statistical Abstract of the United Kingdom* (71st number, pp. 60–63), and in the *Abstracts of Labour Statistics* (1927, pp. 210–13).

The statistics arising from the *National Health Insurance* scheme are clearly exhibited in a paper by Sir A. W. Watson (the Government Actuary) in the *Statistical Journal*, 1927, pp. 433–73.

CHAPTER IX

INCOME AND CAPITAL

1. By *Total National Income* is generally meant the aggregate of the incomes (including earnings) of the persons composing a nation; income is taken as meaning the money, or money value of goods, coming into a person's possession during a year for his own use (subject to rates and taxes), after all expenses connected with obtaining it are subtracted. The earnings of the working-classes, discussed in Chapter VI are thus measured, and incomes are assessed for income-tax on the basis of this definition.

It is doubtful whether a perfectly definite meaning can be attached to Total National Income. The sum of money nominally representing it of course does not actually exist; a great part of income is actually received in the form of cheques which are exchanged for services, and the total is more correctly the total estimated value of services rendered to, or commodities consumed by, the members of the nation, together with the addition to savings, that is to capital goods. In such a total are included the services of an agricultural labourer at £7 per month and of a physician at the same price for a short visit, the value of a week's sojourn at a hotel and the equal value of 180 quartern loaves of bread or 134 oz. of tobacco. The utility of £1 to a person is in general the less the greater his income, and the total utility of all incomes depends on how they are distributed among persons. On the other side, the value of services and commodities depends on the demand for them. In fact, the hundreds of millions of pounds which make the aggregate are not a homogeneous total and cannot be used for processes of averaging without

analysis. To say that the average income of the inhabitants of the United Kingdom was £90 in 1924 is nearly meaningless, except as an arithmetical entity for use in arithmetical processes. The total depends on the existing method, and the momentarily resulting scale, of valuing various services and commodities; the scale is continually changing, and the total would easily be affected, for example, by a redistribution of income by taxation or under a socialistic *régime.*

Nevertheless, the total and resulting averages can be used for comparing total or average income or wages through a period so short that during it no great changes in valuation or in distribution have taken place.

2. The aggregate of the earnings of the wage-earning class is generally estimated by calculating the average annual earnings of men, women and children from the statistics described in Chapter VI, and multiplying these averages by the numbers of persons occupied, as indicated by the census. There is much that is hazardous in this method, but it seems probable that the aggregate of net earnings received by manual workers (including a valuation for payments in kind, etc.), was *circa* 1911 about £800 Mn. and £1,600 Mn. in 1924, annually in the United Kingdom.* This, of course, ignores completely the value of the unpaid domestic work done by women for themselves or their families or relations, and of many other unpaid services.

The aggregate of incomes, not exempted from taxation as less than £160 or 135 per annum,† is estimated from the income-tax returns at about £960 Mn. in 1911 and £2,220 Mn. in 1924, excluding income of wage-earners. This total includes earned and unearned income.

Besides these two sums there are the incomes of those who neither work for wages nor receive as much as £160 † annually (or £135) as incomes. The total was estimated to be between £300 Mn. and £370 Mn. in 1909 by a Committee

* Excluding Southern Ireland in 1924.

† Exemption limit £160 in 1911, and £135 in 1924 after the allowance for earned income is made.

of the British Association (see *Statistical Journal*, Dec. 1910, for the report), at £310 Mn. in 1911, and £270 Mn. in 1924, when the exemption limit was £135, much lower in the scale of incomes than before owing to the general fall in the value of money. This group is termed "Intermediate Income."

The aggregate of incomes of all kinds was thus estimated at about £2,100 Mn. in the year 1911, and £4,200 Mn. in 1924 *; but these totals must be regarded as subject to considerable error, perhaps as much as 10%.

Estimates on a similar basis show—

	Aggregate Income.	Population of United Kingdom.
	£000,000's.	00,000's
1860	750	28,8
1870	1,000	31,3
1880	1,200	34,6
1890	1,450	37,5
1900	1,750	41,2
1908	1,900	44,1
1911	2,100	46,0
1924	4,200	44,9 †

These numbers are rough and uncertain, but they are better than no estimates, and can be used for such purposes as comparing the burden of taxation at different periods.

It must be remembered that the purchasing power of money diminished between 1860 and 1874 (see Chapter IV), increased till 1895, and fell again till 1907, and was much lower in 1924 than in 1911.

3. This and the following three sections relate only to income

* See Bowley and Stamp, *The National Income*, 1924. "Aggregate Income" £4,213 Mn., less sums due to foreigners £49 Mn., making "Disposable Income" £4,164 Mn. Of this, £361 Mn. is transferred in interest on the National Debt and in war and old-age pensions, and is to be subtracted before we get "Social Income," which was therefore £3,803 Mn. "Social Income" is defined as the aggregate of individual and collective incomes, less incomes received by compulsory reductions from other incomes in return for no services or services not rendered in the year in question.

† Excluding Southern Ireland, which is estimated to have received rather less than 4% of the total income in 1911.

brought under review of the Income-Tax Commissioners. The statistical tables in the Annual Reports are full of pitfalls even for the wary.*

The tax is divided into five schedules, lettered A, B, C, D, E. Schedule A includes profits from the ownership of lands and buildings. One-eighth of the assessed value is deducted from the former and one-sixth from the latter for repairs. Schedule B consists of profits from the occupation of land. These profits are assessed from the rental, and were assumed to be one-third of the rental in 1911, but equal to it in 1924. Since a very considerable proportion of farms are rented at less than £135 (the exemption limit in 1924), Schedule B shows only part of the profits of farming. A very small part is assessed under Schedule D, at the choice of the occupiers.

Schedule C contains income from *Government* Securities (Home or Foreign) only. Other income from abroad comes under Schedule D.

Schedule D is an aggregate of all profits from Businesses and Professions. Till 1923 salaries of employees of private firms were included.

Schedule E is made up of salaries and of wages assessed to tax.

The Amounts assessed to tax in 1911–12 and 1924–5 † were as shown on p. 203.

4. The statistics of Gross Income are often quoted as showing the growth of income as a whole, but they include much that is not income, and the deductions have not been made on a uniform plan. It will be sufficient to outline the methods in the two years of the table.

Of the abatements and allowances (p. 204), (*c*) (*d*) (*e*) (*f*) and (*g*) are definitely not income or not British income; (*b*) is not personal income; (*a*) is an odd sum, chiefly of dividends, that is reviewed and exempted and is best merged with Intermediate

* For any close study of Income-Tax statistics it is necessary to use *British Incomes and Property*, Stamp, 1916, and for more recent figures *The National Income*, 1924, Bowley and Stamp, 1927, should be consulted.

† In 1924–5 Southern Ireland is excluded.

Income. When these are subtracted from Gross Income we get an intelligible total, which corresponds to ordinary ideas of personal income above the exemption limit, though, in fact, it includes a sum, which is difficult to estimate, but is not very large, that accrues to clubs, etc. and is not personal. This

	1911–12.		1924–25.	
	Gross Amount. £Mn.	Actual Income. £Mn.	Gross Amount. £Mn.	Actual Income. £Mn.
Schedule A:				
Lands	52	—	50	—
Houses, including sites	224	—	311	—
Other property	1	—	1	—
Total	277	173	362	225
Schedule B:				
Occupation of land	17	5	49	29
Schedule C:				
British Government Securities	14	—	99	—
Other Government Securities	36	—	53	—
Total	50	46	152	136
Schedule D:				
War securities not taxed at source	—	—	91	—
Railways in United Kingdom	46	—	84	—
Dominion and Foreign securities, etc.	69	—	71	—
Manufacture, Mining, Production	456	—	438	—
Distribution, etc.			176	—
Finance, Professions, etc.			461	—
Salaries	28	—	—	—
Total	599	521	1,321	1,016
Schedule E:				
Salaries	127	—	715	—
Wages	—	—	371	—
Total	127	121	1,086	995
Grand total	1,070	866	2,970	2,401

	1911–12. £Mn.
Gross income brought under the review of the Department	1,070
Subtract:	
Exemptions:	
(*a*) Incomes not exceeding £160	59
(*b*) Charities, hospitals, etc.	14
(*c*) Foreign dividends to foreign residents	2
Allowances from Gross Income:	
(*d*) Repairs—lands and houses	43
(*e*) Empty property	7
(*f*) Wear and tear of machinery, etc.	29
(*g*) Other discharges	50
	204
Taxable income: total (*a*) to (*g*) subtracted from gross income	866
Subtract:	
Allowances from taxable income:	
(*h*) Abatements on incomes under £700	128
(*i*) Life Insurance premiums	12
(*j*) Relief in respect of children	5
	145
Income on which tax was received	721

sum was termed " Taxable Income " in the pre-war Reports, and can be identified back for several years on a nearly unchanged definition. The following table shows the results as given in the Reports for the later years, and adjusted as closely as possible to the same definition in earlier years.

Of the allowances made from " taxable " income (*h*) is the development of an earlier system. In 1911–12, abatements were made before the tax was reckoned of £160 when the whole income was less than £400, of £150 for the range £400 to £500, £120 to income £600, and £70 to £700. 834,000 abatements were allowed in 1911–12, and probably a further small number of persons were entitled to them. The abatement figures gave important information about the distribution of incomes in the lower ranges. (*i*) Life Insurance premiums were exempt up to one-sixth of net personal income. (*j*) A reduction of the tax on £10 was allowed for each child of an income-tax payer under 16 years of age.

ESTIMATED AGGREGATE PERSONAL INCOME ABOVE EXEMPTION LIMIT.
[Exemption limit, 1860–76, £100; 1877–93, £150; from 1894, £160]

Fiscal year.	£Mn.	Fiscal year.	£Mn.
1859–1860	254	1887–1888	504
1860–1861	254	1888–1889	518
1861–1862	244	1889–1890	544
1862–1863	273	1890–1891	568
1863–1864	285	1891–1892	569
1864–1865	309	1892–1893	572
1865–1866	326	1893–1894	562
1866–1867	335	1894–1895	553
1867–1868	341	1895–1896	567
1868–1869	344	1896–1897	587
1869–1870	355	1897–1898	611
1870–1871	385	1898–1899	641
1871–1872	399	1899–1900	663
1872–1873	430	1900–1901	695
1873–1874	461	1901–1902	715
1874–1875	482	1902–1903	720
1875–1876	490	1903–1904	732
1876–1877	473	1904–1905	738
1877–1878	476	1905–1906	753
1878–1879	470	1906–1907	764
1879–1880	462	1907–1908	799
1880–1881	468	1908–1909	824
1881–1882	481	1909–1910	822
1882–1883	493	1910–1911	838
1883–1884	507	1911–1912	866 *
1884–1885	505	1912–1913	907
1885–1886	498	1913–1914	951
1886–1887	496		

* The £960 Mn. named on p. 200 above includes also estimates for farmers' profits not taxed, and for evasion of taxation, the income of charities, and some minor items.

Average income has risen about as fast as the exemption limit during the whole period. The number of income-tax payers is not, and cannot be, known directly from the report; it was estimated at about 1,000,000 in 1906. The table just given probably shows the general features of the growth of that part of the national income which is subject to income-tax with fair accuracy, and the rate of growth may accurately be deduced over quite short periods, if no exceptional event occurred in them; but there are many difficulties, some still

the subject of controversy, in such an estimate, which we will enumerate without discussion :—

The amount shown for a year (say 1906–7) is the total income in respect of which the tax was paid or remitted in the year ending April 5 (1907). Under Schedule D more than half (£373,000,000 profits on businesses not otherwise detailed) is assessed on the average profits of the preceding three years (presumably 1903, 1904, 1905), mines (£16,000,000) are assessed on the average of the preceding five years, and about £54,000,000 more on the profits on the preceding year.* The whole assessment for 1906–7 may be regarded as relating to a short period whose centre is the Calendar year 1905; the whole table should be set back, therefore, about a year, and the peculiarities of individual years are averaged away. Thus the high profits in 1907 continued to have effect on the figures till the year 1912–13, for which the Report was published in the autumn of 1914 !

It is generally supposed that greater vigilance and new powers of the surveyors of taxes disclosed from 1907 onwards considerable amounts of income which had hitherto evaded taxation. If this is so, the amounts for years prior to 1907 should be somewhat raised for comparison with 1907 and later years.

It is believed that some part of the income which is received from abroad, and is liable to taxation, successfully evades taxation; naturally this amount can only be guessed. It is not improbable that in the decade preceding 1914 the net of the commissioners has become finer and wider, and that less and less escapes. Actually £80,000,000, paying tax was identified in 1906–7 as income from abroad, and besides this there are other large sums included in Sch. D (52nd Report, pp. 163–5). If less escape than in former times, earlier figures should again be increased for comparison with more recent.

To get the total income above £160 it would be necessary to add an estimate for such income from abroad as escapes,

* From 1927–8 the income is assessed solely on the previous year or date to which accounts are made up

and also an estimate for profits of trades and professions which are generally believed to be on the whole under-valued. £80,000,000 was a guess current *circa* 1907 for these two amounts together, but in fact there are practically no data for an estimate.

5. Sometimes the gross returns for income-tax have been placed alongside the returns of Changes of Wages, discussed in Chapter VI above, and the conclusion drawn that income grew continuously while wages have been nearly stationary from 1900–8 to 1913. Wages did in fact lose relatively to incomes in this period, but the relative rates of growth cannot be shown from the statistics, for the following reasons :—

The wage-changes published only apply to a small part of the working population, and afford no test of the general growth of wages (pp. 159 *seq.*, above).

The most recent statistics available were for the income assessed for 1912–13 (and even these are incomplete), and these belong to 1911 rather than to any other year.

The income-tax returns cannot be allotted to any one year.

The relation of gross to net income has changed.

The collection of the tax has recently been more thorough.

The total net income, as shown in the table above (p. 205), *naturally grew* 1% *per annum* with population, while the wage-changes have no relation to population.

The year 1900, which is frequently taken for comparison, was a year of exceptional inflation for wages, but is a normal year in the income-tax returns.

The following table shows the present writer's estimate of the change of average wages, and of the average income of the income-tax payer, for the period 1880 to 1912 each expressed as a percentage of the level in 1880. The former is from p. 165 above, the latter is based principally on the method of the table * on p. 205, with the years adjusted and allowance made for the growth of population, and with some

* See *Economic Journal*, 1904, p. 459; and for another view of the same problem, see *The Change in the Distribution of Income*, Clarendon Press, 1920, by the present author.

other modifications based on the discussion in paragraph 5 above. The method is open to a great many fairly obvious criticisms.

INDEX-NUMBERS OF INCOMES AND WAGES †

	Wages.	Incomes.		Wages.	Incomes.		Wages.	Incomes.
1880	100	100	1891	115	103	1902	126	118
1881	100	100	1892	115	100	1903	125	118
1882	103	103	1893	115	100	1904	123	119
1883	103	101	1894	115	101	1905	123	120
1884	103	100	1895	115	103	1906	126	124
1885	101	97	1896	115	107	1907	133	127
1886	100	97	1897	116	109	1908	131	125
1887	101	99	1898	120	111	1909	129	127
1888	104	103	1899	123	113	1910	129	131
1889	110	107	1900	130	117	1911	131	133
1890	114	105	1901	128	117	1912	135	138

† There is no allowance for the cycle of unemployment in this table. Such allowance would raise the numbers in some years and lower them in others, without affecting the general run. The averaging of the incomes under Schedule D also merges together good and bad years for the income index-number.

There is no allowance for the cycle of unemployment in this table. Such allowance would raise the numbers in some years and lower them in others, without affecting the general run. The averaging of the incomes under Schedule D also merges together good and bad years for the income index-number.

In making this computation care has been taken to exclude the same *proportion* of income as exempt (see *Economic Journal*, 1904, p. 460), so that the intermediate class who are not wage-earners but have small incomes are excluded from the calculation on the same proportionate basis throughout.

6. We now come to the system of assessment in 1924–5. From the Gross Income, £2,970 Mn., exemptions and allowances as (*a*) to (*g*) 1911–12 amounting to £569 Mn. are subtracted, the remainder, £2,401 Mn., is termed "Actual Income." Next, one-sixth of what is defined as earned income is subtracted up to a maximum of £250 (on £1,500 income) for an individual.* In 1924–5 £127 Mn., was so deducted, leaving

* In earlier years one-tenth was subtracted up to a maximum £200 or £2,000 income. In 1911 the tax on earned income up to £2,000 was

£2,274 Mn. which is termed "Assessable Income." Next, instead of abatements such as (*h*) in 1911–12, a personal allowance of £135 is made for a single man or woman and of £225 for a married couple, and instead of (*j*) a considerably larger allowance for children and some other dependents. The remainder, £1,349 Mn., is called "Taxable Income," a use of the term which differs from that in the pre-war reports. One-half the standard rate is then imposed on the first £225 of an individual's taxable income, and the full rate on the remainder, with an allowance for Life Insurance premiums.

The result of these allowances, etc. was to reduce the average tax per £ of "Actual Income" to half its standard rate—to 2*s.* 3*d.* instead of 4*s.* 6*d.* A married wage-earner with one child became completely exempted if his receipts per quarter were less than about £75, so that, in fact, only a very small proportion of wage-earners (such as bachelors earning over 63*s.* weekly) paid any income tax at all.

Under all the schedules together, the estimated number of individuals with incomes over £135 per annum was 4,600,000, of whom 2,300,000 were entirely exempt from income-tax. The great increase in the number above the exemption limit over the number in 1906 (p. 205) is partly due to the lowering of the limit, but mainly due to the great rise in money wages and salaries corresponding to the rise of prices.

7. An interesting item in recent reports is the record of the number of new houses assessed under Schedule A each year.

NEW BUILDINGS ASSESSED, GREAT BRITAIN

Dwelling-houses, including Flats

Annual value.	1921–22.	1922–23.	1923–24.	1924–25.
Under £20 . .	20,730	40,559	25,334	27,778
£20 to £40 . .	44,799	81,149	47,452	55,232
£40 to £60 . .	5,307	8,405	10,298	14,995
£60 and over . .	1,811	2,610	5,159	7,885
Total . .	72,647	132,723	88,243	105,890

9*d.* in the £, and on "unearned" 1*s.* 2*d.* This differentiation was made in assessing the tax, not as in 1924–5 by lowering the assessed income and then assessing a uniform tax.

8. Since the abolition of the system of abatements we have no longer any means of describing the distribution of incomes under £2,000 per annum, but the Super-tax statistics afford a nearly complete account of incomes above that amount. In 1911 the lower incomes for super-tax was £5,000.

SUPER-TAX. DISTRIBUTION OF INCOMES

Class.	United Kingdom. 1911–12.		Great Britain and Northern Ireland. 1924–25.	
	Number of persons.	Total Income assessed. £Mn.	Number of persons.	Total Income assessed. £Mn.
£2,000–2,500	—	—	21,341	47·8
£2,500–3,000	—	—	14,650	40·1
£3,000–4,000	—	—	17,487	60·2
£4,000–5,000	—	—	9,797	43·7
£5,000–10,000	8,049	54·3	16,940	115·7
£10,000–20,000	2,899	39·1	6,263	85·1
£20,000–50,000	1,100*	45·3	2,378	69·1
£50,000–100,000	183*		421	28·2
£100,000 and over	68	12·5	138	28·0
Total	12,299	£151·2	89,415	£517·9

Again the change in the value of money must be remembered. The number in 1924–5 may be slightly increased by late assessments.

9. The aggregate capital owned by the individuals of the nation can be estimated either by capitalizing the "unearned" income, or from the records of estates paying death duties. The first method was used by Sir R. Giffen in his essay on "Recent accumulations of capital in the United Kingdom," † 1878. The latter has been the subject of much recent work. Unfortunately, it is extremely difficult to reconcile the results reached by the two methods.

To use the records of estates, assessed for Estate Duty, it is necessary to estimate the number of estates in existence in relation to the number which pass per annum. The best

* Approximate.

† *Essays in Finance.* Also in *Statistical Journal*, 1878.

estimate appears to be that by Sir B. Mallet, and Mr. H. C. Strutt,* who, by tabulating the values of the estates according to the age of the deceased, and multiplying by the reciprocal of the death-rate age by age, arrive at the multiplier 30; that is, they conclude that 30 times the value of estates passing in one year gives the total value of such estates in existence. A higher multiplier had been used in previous estimates, but this neglected the important fact that estates as a whole increase with the age of their possessors.

The table below shows the results of this estimate in relation to the income-tax returns. It is modified from that on p. 220 of the Report of the Committee on the Income Tax.†

It is evident that the rates of interest shown in this table are higher than those in fact obtained. Indeed, in the

UNITED KINGDOM

Assessed values of estates reviewed for estate duty. Average of 10 years, 1894–1904, multiplied by 1·1 to bring up to 1904–5.		Presumed assessed value of all estates, 30 times previous column.	Corresponding income 1904–5, from income-tax returns. [Allowances deducted from gross income, but not insurance, abatements or exemptions.]		Average deduced from previous columns.	
					Rate of Interest per cent.	Number of year's purchases.
	Millions.	Millions.		Millions.		
Stocks, companies, mortgages, bonds, mines and quarries . .	£127	£3,810	Companies, etc.	£265	7·0	14¼
Agricultural land, timber, building land .	26	780	Lands . .	41	5·3	19
Houses, and all rents that can possibly be connected therewith .	63	1,890	Buildings .	153	8·1	12
	£216	6,480		£159	7·1	14·1
Goodwill, share in firms, book debts, stock-in-trade, half cash at bank	29	870	Unknown.			
	£245	7,350				
Insurance, debts, small sundry properties, personal goods, half cash at bank . . .	35	1,050	No corresponding income.			
	£280	8,400				

* *Statistical Journal*, July 1915, "The Multiplier and Capital Wealth." The multiplier 28 is obtained in the main analysis, but it is subsequently raised to 30 (p. 596). † H.C. 365 of 1906.

Report of the Commissioners (Cd. 2633),* the net income from lands is stated at 4·3% (instead of 5·3%), and from buildings at 5·5% (instead of 8·1%). In the paper alluded to a searching examination is made of the reasons of this discrepancy, and it is found that in 1912–13 the taxable income arising from property is only 6·6% (instead of 7·1 as above) of the corresponding capital so estimated; the gap between the results of the two methods is thus reduced to one part in 14, and in view of the difficulties in both methods it is perhaps not greater than is to be expected.

The method of capitalizing income is the one adopted by Sir J. Stamp (*British Incomes and Property*, p. 404). He arrived at the total £14,300 Mn. for the capital value of private and governmental property in the United Kingdom in 1914, but considered that the range of doubt was as great as 13%.

10. It is probable, however, that the increase of the total value of estates liable to duty observed over a period long enough to eliminate the accidents of individual years has a close relation to the growth of capital. The table on p. 213 shows these values since the commencement of the duty.

In the first two years the totals are those of the capital on which duty was *paid*, which is less than the capital *liable* to duty which is that shown for the other years, since the payment is in some cases made in instalments.

Hence the capital thus passing in 1925 was 64% more than in 1911, or, allowing for the exclusion of South Ireland and the possible increase in gifts "inter vivos," perhaps 75%. The growth of income of all kinds in this period, however, was 100% (p. 201).

11. Income in the United States was estimated in 1920 by the National Bureau of Economic Research.† Two methods were employed, which were to a considerable extent independent of each other. The first, called "Estimate by Sources of Production," is based on the material provided by the

* See also the table, pp. 80–1, in the 52nd Report (Cd. 4226).

† *Income in the United States, Its Amount and Distribution*, 3 vols., New York, Harcourt, Brace & Company, 1921. See Vol. 1, p. 13, for the figures quoted.

Net capital value of estates which become liable to estate duty.	Estate and other duties paid.	No. of millionaires included.
	United Kingdom.	
	£000,000's	£000,000's	
1895–96	213	14	8
1896–97	219	14	5
1897–98	247	15	7
1898–99	251	16	9
1899–00	293	18	12
1900–01	265	17	9
1901–02	289	19	8
1902–03	270	18	4
1903–04	264	17	7
1904–05	265	17	1
1905–06	272	17	8
1906–07	298	19	10
1907–08	282	19	7
1908–09	271	18	9
1909–10	284	22	5
1910–11	273	25	14
1911–12	278	25	5
1912–13	279	25	11
1913–14	296	27	11
1914–15	307	28	8
1920–21	391	48	11
1921–22	420	52	11
	Great Britain only.		
1922–23	431	57	15
1923–24	442	58	9
1924–25	461	59	13
1925–26	456	61	7

Census of Production, and a similar method has been used in the United Kingdom in the Report of the Census of Production of 1907. The second, "Estimate by Incomes Received," is generally similar to that outlined on p. 200 above. In both countries the large part of income which is received for services, etc. not directly connected with material production can be estimated only by the latter method, which is used in the following Table:

ESTIMATES OF NATIONAL INCOME

	1911.	1919.
United States:		
Aggregate .	$31,200Mn.	$66,000Mn.
Per head .	$337	$629
United Kingdom:		
Aggregate .	£2,100Mn.	£4,200Mn.
Per head .	£46	£93

It is not possible to make any valid estimate for income in the United Kingdom in 1919, but no doubt one can be made for that in the United States in 1924.

The methods and results of estimating both Income and Capital in a number of countries in 1914 are described by Sir. J. Stamp in the *Statistical Journal*, 1917, pp. 441 *seq.*

CHAPTER X

TAXES AND RATES

1. SUMMARY statements of National Revenue and Expenditure are to be found in the *Statistical Abstract of the United Kingdom*. They involve many difficulties of definition and interpretation, and should be studied in conjunction with the *Annual Finance Accounts of the United Kingdom* (*e.g.* H. of C. 71 of 1927), and with the Reports of the Commissioners of Inland Revenue and of Customs and Excise.

Throughout the statistics of this Chapter the changes in the purchasing power of money must be borne in mind, as shown in Chapter IV. In particular, post-war and pre-war totals should never be compared without reference to the general rise in prices and increase in income.

The following tables for the Fiscal Year beginning April 1st, 1926, are compiled from the *Statistical Abstract* with some modifications of arrangements.

IMPERIAL REVENUE OF THE UNITED KINGDOM, 1925–6

Exchequer Receipts, £Mn.

Customs			103·5
Inland Revenue:			
Excise	134·6		
Stamps (excluding Fee and Patent)	24·7		
Land Tax	0·7		
Land Value Duties	0·2		
Income-tax	259·4		
Super-tax	68·5		
Estate (Death) Duties	61·2		
Corporation Profits Tax	11·7		
Excess Profits Duty *	2·0		
Motor Vehicles Duties	18·1		
Total Inland Revenue		581·1	
Total Revenue from Taxes			**684·6**

Postal, Telegraph, Telephone :		
Net receipts		3·4
Crown Lands	1·0	
Receipts from Sundry Loans	14·9	
Total from Property		15·9
Miscellaneous Receipts :		
Ordinary	17·4	
Special	36·9	
Total		54·3
TOTAL net REVENUE		758·2

IMPERIAL EXPENDITURE OF THE UNITED KINGDOM, 1925–6

Exchequer Issues, £Mn.

National Debt Services :		
Interest to United States	28·3	
Other Interest	278·7	
Management and Expenses	1·2	
Sinking Fund	50·0	
Total		358·2
Defence :		
Army	44·2	
Navy	59·7	
Air Force	15·5	
Total		119·4
Civil List and other miscellaneous expenses charged on Consolidated Fund		3·0
Civil Services :		
I. Public Works	6·9	
II. Salaries, etc., not in other classes	10·8	
III. Law and Justice	11·8	
IV. Education, Science and Art	48·6	
V. Foreign and Colonial	7·6	
VI. Non-effective and Miscellaneous	99·0	
VII. Health, Labour and Insurance	36·4	
Unclassified	22·2	
Total		243·3
Customs, Excise and Inland Revenue Departments		11·4
Payments to :		
Road Fund	17·5	
Northern Ireland Exchequer	4·9	
Local Taxation Accounts	14·5	
Total		36·9
TOTAL net EXPENDITURE		772·2
Excess over net Revenue		14·0

The total stated in the Abstract for the Revenue is £812 Mn. instead of the £758 Mn. in the table above. The difference illustrates the difficulty of defining the Revenue. The official account includes the whole receipts from the Post Office (£57·4 Mn.), while it is only reasonable to deduct the expenses of conducting the postal, telegraph and telephone services (£54 Mn.), and enter the balance as in the table. Even then it is doubtful whether the balance (£3·4 Mn.) is a tax or a trading profit. It may also be argued that since motor vehicle duties, except for £600,000 annually in lieu of former carriage duties, are allotted to the Road Fund (apart from occasional raids on the fund by the Chancellor of the Exchequer), they also should be entered net, and in the table on p. 220, this has been done in order better to preserve comparability with pre-war figures. The amount actually raised by taxation is thus £685 Mn. (or, subtracting the issues to the Road Fund, £667 Mn.), and not the total revenue £812 Mn. which is commonly spoken of as obtained from taxes.

The profits from the Post Office and the receipt from Crown Lands and from Sundry Loans, etc., are properly included in Revenue. The details of the Loans, etc. were as follows :—

RECEIPTS FROM SUNDRY LOANS, ETC., 1925–6

	£Mn.
Suez Canal Shares	1·1
Anglo-Persian Oil	0·6
Interest on sundry advances	1·5
Interest on War Loans : To Empire	6·3
To Allies	4·5
Interest on Reconstruction Loans	0·9
	14·9

On the other hand, it is doubtful whether a great part of the miscellaneous receipts ought to be counted as Revenue. Part is for services rendered by the Civil Departments, part from interest on special funds, and part from sales of property. On the whole, the sums listed under Ordinary Receipts are revenue and under Special Receipts are not (p. 218).

The sums from Sundry Loans and Miscellaneous Revenue can to some extent be placed against the payment of Interest

MISCELLANEOUS REVENUE, 1925–6

	£Mn.
Ordinary Receipts:	
Currency Note Investment Fund	5·9
Interest accrued in Post Office and Trustee Savings Banks	1·9
For various sales, rents, services	2·8
Fee and Patent Stamps	1·9
Receipts for various services not directly appropriated to expenditure	3·4
Miscellaneous earnings and receipts	1·5
Total	17·4
Special Receipts:	
From Reparations	10·1
„ Enemy Debts realization	10·0
„ Surplus from Food Commission	—
War Risks Insurance, etc.	3·0
„ Disposals and Liquidation Commission	7·2
Other sums realized from sales, etc.	5·5
Contributions to war cost and repayments	1·1
Total	36·9
Total Miscellaneous Revenue	54·3

to the United States (£28 Mn. in 1925–6) included in the Expenditure on National Debt Services, and the Sinking Fund by which the National Debt is reduced.

Of the Expenditure, about half is included under "Consolidated Fund Services," which do not require to be voted each year by the House of Commons, while the estimates for the remainder must be agreed annually in detail. The Consolidated Fund Services include National Debt Services, the Road Fund, Payments to Local Taxation Accounts to supplement Rates,* Payments to Northern Ireland, in consequence of the agreement made when its Administration was delegated under the Act of 1920, and sums (£780,000 in 1925–6) allotted to the Land Settlement scheme for ex-service men. The Consolidated Fund Services are completed by the Civil List (the agreed income transferred to the Royal Family and others), Judges' Salaries, Civil List Pensions, and other items—in all £2·4 Mn. in 1925–6.

* These, in fact, are only part of the sums transferred to Local Authorities. Other parts, amounting to £72 Mn., are contained in Classes III, IV, VII, etc.

The Civil Services are grouped in seven classes, as shown in the table. Class VI, non-effective and miscellaneous, consists chiefly of war pensions and old age pensions, about £67 Mn. and £27 Mn. respectively. Class VII includes the national contribution to the Unemployment and Health Insurance Funds, and the expenses of the Ministries concerned.

The whole classification was under revision in 1927. "Unclassified" includes occasional expenses, such as those resulting out of the War, and against them in such years as 1920–21, 1921–2 (table, p. 220) are to be put receipts from realization of war-stores. In 1925–6 the main item was the subsidy to the Coal Industry (£19 Mn.), which resulted in an excess of expenditure over revenue in that year. When realized revenue exceeds realized expenses the excess is used in redemption of the National Debt; in the opposite case the deficit is carried to the debt. Normally, a definite sum, £50 Mn. in 1925–6, £60 Mn. in 1926–7, is set aside each year in the estimates for debt redemption as a sinking fund. The balance is not, in any case, carried forward to the next year's accounts.

It should be noticed that the management of the Debt (£1·2 Mn.), and the expenses of the collecting departments, customs, etc., (£11·4 Mn.) form a very small percentage of the revenue or expenditure.

2. Similar statements for other years are given in the table on p. 220. Since the form of the published accounts varies from time to time, some adjustments have been necessary. It must be remembered that Southern Ireland was separated from the United Kingdom in 1923, and that therefore statistics before and after that date are not strictly comparable.

In the lower part of the table, Class IV of the Civil Service (Education) is shown separately. Classes VI and VII are merged into "Pensions, Labour, Health, Insurance."

Clauses I, II, III, V, the Civil List, etc., and the expenses of the Customs, Excise and Inland Revenue department are merged into "other normal services."

National Revenue and Expenditure, United Kingdom (£Mn.)

	1904–5.	1908–9.	1913–14.	1920–1.	1921–2.	1922–3.	1923–4.	1924–5.	1925–6.	1926–7.	1927–8.†
Revenue:											
Customs and Excise	72·0	62·9	75·0	333·8	324·3	280·3	267·9	234·5	238·0	240·5	257·9
Income and Super-tax	31·3	33·9	47·2	394·1	398·8	379·0	330·0	336·5	327·9	300·6	309·0
Estate Duty, etc.	16·7	18·4	27·4	47·7	52·2	56·9	57·8	59·4	61·2	67·3	67·8
Excess Profits and Corporation Duties	—	—	—	219·8	48·0	21·0	23·3	18·8	13·7	8·5	5·7
Other Taxes	10·4	10·5	13·4	29·2	22·3	25·2	24·4	24·3	25·7	25·6	26·3
Post Office, profit or loss	4·7	4·2	6·2	−4·2	−9·6	3·3	3·0	5·1	3·4	4·0	4·4
Motor Vehicles duty, less Road Fund	—	—	—	−1·9	0·3	0·6	0·6	0·6	0·6	4·0	4·6
Crown Lands and Sundry Loans	1·5	1·7	2·1	31·4	14·6	10·9	13·5	12·9	15·9	23·9	24·5
Miscellaneous Revenue	1·4	2·0	2·3	313·3	197·1	75·2	52·8	41·4	54·3	59·1	57·5
Total	138·0	133·6	173·6	1,363·2	1,048·0	852·4	773·3	733·5	740·7	733·5‡	757·7
Expenditure:											
National Debt Services	27·0	28·0	24·5	349·6	332·3	324·0	347·3	357·2	358·2	378·6	370·0
Defence	66·1	59·0	77·2	293·2	189·4	111·0	105·8	114·7	119·4	116·7	115·1
Civil Service and Administration:											
Education	15·6	17·4	19·5	59·3	65·4	49·8	47·7	48·7	48·6	53·2	
Pensions, Labour, Health, Insurance	0·7	2·9	20·8	76·3	178·0	142·0	131·2	135·1	135·4	134·6	251·5
Other normal services	12·8	17·4	19·8	67·3	99·3	57·7	60·7	46·7	50·8	54·1	
Unclassified	—	—	—	269·4	123·1	51·3	13·4	9·0	22·2	13·0	
To Local Authorities, and Northern Ireland	14·3	9·8	9·7	10·8	12·3	13·8	17·6	17·8	19·3	19·9	19·7
„ Land Settlement	—	—	—	6·9	2·6	1·2	1·2	0·7	0·8	—	—
Total	136·5	134·5	172·9*	1,132·8	1,002·4	750·8	724·9	729·9	754·7	770·1	756·3

* Including 1·4, expenditure on Road Fund.
† Estimates, April 1927.
‡ This deficit is mainly attributable to the Coal Stoppage in 1926. The receipts from Income Tax were £20 Mn. below the estimate, and from Excise £8 Mn. below.

3. Details of the receipts from Customs and Excise in selected years were as follows :—

	1904–5.	1913–14.	1925–6.
Customs :	£Mn.	£Mn.	£Mn.
Tobacco	13·2	18·3	53·5
Tea	8·3	6·5	5·8
Spirits	4·0	4·4	7·9
Wine	1·2	1·2	3·7
Beer	—	—	6·1
Matches	—	—	1·7
Sugar	6·1	3·3	18·4
Silk (including artificial)	—	—	2·6
Safeguarded Industries	—	—	2·0
Key Industries	—	—	0·5
Coal export	2·0	—	—
Others	1·1	1·7	1·3
Total	35·9	35·4	103·5
Excise :			
Spirits	18·1	19·5	42·0
Beer	13·1	13·6	76·3
Sugar	0·1	0·1	1·0
Matches	—	—	1·6
Artificial Silk	—	—	0·6
Entertainments	—	—	5·7
Licences	4·3	5·7	5·0
Other	0·4	0·7	2·3
	36·0	39·6	134·5

4. The Inhabited House Duty, repealed in 1924, was specially interesting for the statistician, for in the tables relating to it (*e.g.* 52nd Report of the Commissioners of the Inland Revenue, pp. 113 *sqq.*) we had information as to the assessed value of all the inhabited houses and residential shops and premises, and in less detail of uninhabited premises, in England, Wales and Scotland. The duty was not imposed in Ireland. The tables on pp. 222–3 show the nature of the information. The first and third were discontinued after 1913–14, the second after 1914–15.

GREAT BRITAIN, 1907-8 *

Exempt from duty.	No. of premises.		Annual value.	
			£000's	
Premises not used as dwellings	664,266		49,819	
† Separate dwellings exempt from duty	64,681		845	
Royal and diplomatic residences, hospitals, schools, etc.	33,872		4,089	
Houses of annual value—				
under £10	3,162,752	6,112,736	20,130	59,966
£10 and under 15	1,985,639		23,463	
15 and under 20	964,345		16,373	

Charged to duty.	No. of premises.		Annual value.	
	Private dwelling houses.	Others.‡	Private dwelling-houses.	Others.‡
			£000's	£000's
†"Separate dwellings"—				
£20 and under £41	19,261	—	486	—
41 „ 61	4,695	—	234	—
Houses—				
£20 and under £25	369,640	85,219	8,069	1,820
25 „ 30	248,531	65,183	6,595	1,708
30 „ 41	402,454	123,662	13,818	4,248
41 „ 50	103,352	34,115	4,600	1,532
50 „ 61	123,072	50,144	6,646	2,735
61 „ 80	61,151	29,821	4,195	2,079
80 „ 100	38,245	20,919	3,300	1,812
100 „ 150	44,581	22,435	5,227	2,625
150 „ 200	16,468	9,154	2,733	1,514
200 „ 300	13,460	7,138	3,137	1,670
300 „ 400	5,199	3,122	1,725	1,040
400 „ 500	2,370	1,531	1,024	664
500 „ 1000	2,826	2,328	1,827	1,507
1000 and over	970	836	2,093	1,870
	1,456,275	455,607	£65,710	£26,825

* The statistics are subject to slight additions when arrears are collected.
† That is, parts of buildings (*e. g.* flats) used as separate dwellings.
‡ Residential shops, hotels, public-houses, etc., farmhouses, lodging-h

Great Britain

Number and Value of Premises charged to Duty

Class.	Private dwelling-houses.*				Other premises.†	
	Number.		Annual value.		Number.	
	1907–8.	1914–15.	1907–8.	1914–15.	1907–8.	1914–15.
	000's.		£000,000's.		000's.	
£20 to £41 . .	1,045	1,167	29·2	32·4	274	281
41 „ 61 . .	226	236	11·3	11·7	84	83
61 „ 80 . .	61	63	4·2	4·3	30	30
80 „ 100 . .	38	39	3·3	3·4	21	20
100 „ 150 . .	44·5	45	5·2	5·3	22	21
150 „ 200 . .	16·5	17	2·7	2·8	9	8
200 „ 500 . .	21·0	21	5·9	5·9	12	9
500 „ 1000 . .	2·8	2·8	1·8	1·8	2·3	1·5
£1000 or more .	1·0	0·9	2·1	2·0	0·8	0·7
	1,456·0	1,592	65·7	69·6	455	454

	Nos. of private dwelling-houses, whether charged to or exempt from duty.			
	Metropolis.	Rest of England.	Scotland.	Great Britain.
	000's	000's	000's	000's
"Separate dwellings" exempt	54	11	—	65
„ „ £20 to £61	21	3	—	24
Houses to £10 . .	7	2,580	575	3,163
£10 „ 15 . .	42	1,765	178	1,986
15 „ 20 . .	73	803	89	964
20 „ 25 . .	92	248	30	370
25 „ 30 . .	56	169	24	249
30 „ 41 . .	125	243	34	402
41 „ 100 . .	108	188	29	326
100 „ 500 . .	27	49	6	82
500 . . .	3	1	—	4
	608	6,060	965	7,634

* Including parts of buildings used as separate dwellings.
† Residential shops, hotels, etc., lodging-houses, farm-houses.

The importance of these statistics was in their relation to the social grading of the people, a subject with which the population census does not deal, and also in relation to the statistics of income. The income-tax returns and the value of houses cannot easily be compared, but there is here a possible field of investigation of a difficult character. In general (but with many exceptions) there is one private dwelling-house to one payer of income-tax, and also in general (but with some extraordinary exceptions) the higher the income the larger the value of the house occupied, but the smaller the proportion of income spent on rent; this proportion probably varied from 25% for some classes of workmen in the large towns to 10% for persons with an income of £700 a year. The number of income-tax payers in Great Britain was probably a little less than the aggregate number of houses of value above £30 in London and above £25 in the rest of Great Britain. The aggregate annual value of these houses was about £55,000,000 in 1907; the aggregate income of income-tax payers in Great Britain was somewhat over £600,000,000. There is much that is hypothetical in this comparison, but it suggests an interesting line of analysis.

5. From the table of expenditure on p. 216 above, it is clear that Local and Central Expenditure cannot be separated from each other; and though rates and taxes are generally paid to different authorities, they are equally a compulsory drain on the pockets of the payer. We will, therefore, investigate the total sum expended locally in selected years in England and Wales (p. 225). The tables in the Statistical Abstract on Local Finance need careful interpretation, and it is not easy to combine Scotland and Ireland with England and Wales, especially since 1921.

It is not practicable without a long investigation to allot the whole of this £393,000,000 to categories of expenditure, and there are innumerable cross-accounts with the Central Government, with Capital and Interest balances, with municipal trading undertakings, and with the allotment of par-

LOCAL AUTHORITIES, ENGLAND AND WALES

	1904–5.	1913–14.	1923– 4.
Receipts :	£Mn.	£Mn.	£Mn.
Public Rates	56·0	71·3	143·3
Government Contributions	19·6	22·6	78·3
Tolls, Dues and Duties	4·3	8·5	15·8
Water, Gas, Electric Light, Tramways and Light Railways	19·5	33·1	73·7
Repayment for private improvements	1·8	1·3	1·3
Miscellaneous Receipts	9·0	12·5	37·1
From Loans	33·4	20·0	46·5
Total	143·6	169·3	396·0
Expenditure (including loan charges) :			
Education	21·9	31·8	72·3
Poor relief	11·5	12·3	32·5
Hospitals and Asylums	5·1	6·7	14·5
Highways, Markets, Harbours, Public Lighting and Sewerage	24·3	32·6	69·8
Police	6·1	7·7	18·8
Libraries, Parks	1·8	2·3	5·3
Gas, Water, Electric Lighting, Trams, etc.	18·7	32·9	71·5
Housing	—	0·6	16·3
Small Holdings	—	0·5	2·4
On private improvements	1·9	1·3	1·4
Other	16·4	19·6	38·5
Total defrayed from Revenue	107·7	148·3	343·3
Defrayed from Loans	31·4	21·1	50·0
Total	139·1	169·4	393·3

ticular receipts to particular purposes. The table is given mainly for the purpose of exhibiting the difference between the sum drawn in rates and total receipts, and to show the increase in 20 years.

6. We can now bring together, at least in an approximate way, the total of the compulsory payments in rates and taxes and of other public receipts (p. 226).

The total sums received by Local Authorities are very much greater owing to transference to them from National Receipts.

From the Income Statistics given on p. 201 above we can

PUBLIC RECEIPTS, UNITED KINGDOM (£Mn.)
(Southern Ireland excluded in the last column)

	1904–5.	1913–14.	1923–4.
NATIONAL RECEIPTS.			
Indirect Taxation :			
Motor Vehicles Duties . . .	—	—	15
Customs, Excise, Stamps, etc. .	78	85	289
Direct Taxation :			
Income and Super-tax, Excess Profits, Corporation Profits and Land Taxes, House and Land-value Duties	51	78	415
Total from taxes . . .	129	163	719
Profits from Post Office . . .	5	6	3
Crown Lands, Suez Canal, Loans, etc.	2	2	14
Total National Receipts .	136	171	736
LOCAL RECEIPTS.			
Rates	65	82	163*
Total Receipts . . .	201	253	899

* Only a rough approximation for the relatively small totals of rates in Scotland and Northern Ireland is included.

obtain some idea of the relation between the National Income of the United Kingdom and the amount appropriated in taxes and rates. Taxes and rates expressed, approximately, as a percentage of income were: in 1860, 12%; in 1880, 9%; in 1890, 8%; in 1894, 9%; in 1904, 11%; in 1913, 12%, and in 1924, 22%. It is highly probable that the percentage fell from 1860 to 1890, and that the grant of old age pensions, and the expenses of the National Health Insurance Act, brought it in 1913 up to the same figure as in 1860. To find out what part of the increase since 1913 is due to the War, and what part to increased social services, would necessitate a very troublesome analysis.

APPENDIX I

EXERCISES

[References are to tables in the preceding pages, or to the *Seventy-first Statistical Abstract for the United Kingdom*, 1912 to 1926. Cmd. 3084. Price 6*s.* 6*d.*]

PART I

On Chapter II

1. Write down the number of bushels (p. 6) in the forms (*a*) to (*f*).

2. Add together 75,324, 79,476, 432,132, the numbers being correct to 1%, 2%, 3% respectively.

3. The population of a colony consists of 73,243 Europeans, $7{,}8^{00}$ Indians and 432^{000} negroes. What is the whole population?

4. The average wage of $2{,}456^{000}$ workmen is 46*s.* 6*d.* (to nearest 6*d.*). What is their aggregate wage?

5. The total income of $3{,}254{,}6^{00}$ persons is £243 $\times$ 10^6. What is the average income?

6. The productivity of $4{,}325^{000}$ acres is between 38 and 39 quarters of wheat per acre. A quarter weighs between 470 and 490 lbs. What is the yield in tons?

7. £87,547 is to be raised in rates, where the assessed annual value is £943,650. Find the least rate necessary (fractions of one farthing not being used) and the excess collected.

8. The quantity of wheat imported in 1894 was $70{,}126^{000}$ cwts., in 1908 $91{,}131^{000}$ cwts. Express the ratio in the notations of pp. 11, 12.

9. If wages per hour rose 20%, and the number of hours worked per week fell 10%, find the change in weekly wages.

10. Wages were raised 10%, lowered 15%, raised 20%, lowered 25%, and raised 10% in certain years, each percentage being reckoned on the wages current when the change was made. Find the change in the whole period.

11. Express the total imports in Class I, Class II, Class III and Classes IV and V as percentages of the grand total in each of the years 1914, 1921 and 1925. (*Stat. Abs.*, Table 213, pp. 310–11.)

12. In the same table express total exports of United Kingdom produce in each year from 1912 to 1922 as percentages of the total in 1913.

On Chapter III

1. From *Stat. Abs.*, Table 161, pp. 232–3, find the average value per cwt. in 1925 of each of the kinds of fish of which the aggregate value exceeded £200,000.

2. Find the average production per acre of the 6 corn crops shown in Tables 156 and 157 for (1) Great Britain, (2) Ireland, for the years 1912 and 1922.

		Acreage.	Average yield per acre.
3.	A.	3,456,789	35·2 bushels
	B.	2,703,257	30·7 ,,
	C.	1,432,843	43·8 ,,

Find the average yield in A, B, C together by the methods of p. 18.

4. Using the methods of p. 20, check the averages shown in the tables in Part I, Ch. VI, p. 51, and in Part II, Ch. VI, p. 168, suggesting the cause of the discrepancies (if any) found.

5. Find the arithmetic average, the median, the quartiles and the mode of the miners' ages given in Part I, Ch. V, p. 36.

6. Find the average prices of the four kinds of woollen and worsted tissues distinguished in *Stat. Abs.*, Table 220, pp. 348–9, per linear yard in 1913 and per square yard in 1926.

At each date express the prices of a yard of the last three as percentages of the price of the first-named kind.

7. Criticize the following averages: Table 115, p. 171. Total spent in poor relief, 1923–4, England and Wales: £32,464,401. Total relieved (p. 65): July 1st, 1923, 1,385,978; Jan. 1st, 1924, 1,372,098. Average number relieved, 1,379,038. Average cost, £23 10*s.* 10*d.*

8. If the average wage of 55,000 men is 48*s.* 6*d.*, and of these the average for 30,000 is 45*s.*, find the average for the remainder.

9. Find d_1, d_9 (the 1st and 9th "deciles"), in the table on p. 23, so that "one-tenth of the wage-earners received d_1/- or less, and one-tenth received d_9/- or more."

On Chapter IV

1. Write the net value of Consignments (*Stat. Abs.*, Table 209, pp. 296–7) from the 34 British Countries for 1912 and 1925 in 6 columns as in the table in Part I, Ch. IV, p. 27, and calculate the ratios of the totals. Make two new columns also showing the values to the nearest million £, and calculate the ratios.

Find also the relative and the absolute errors in the totals of each of the columns and comment on the results.

2. Write the table of monthly prices of wheat (*Stat. Abs.*, Table 159, p. 224) for the years 1913, 1920, 1926, (1) omitting pence, (2) to the nearest shilling. Find the averages for the years, and express them as percentages of the average found for 1913. Comment on the result.

3. Re-write the table showing the percentage of unemployed monthly from 1897 to 1906 (Part II, Ch. VII, p. 179), (1) omitting the decimals, (2) to the nearest whole numbers. Calculate the yearly means and the 10 years' monthly averages. Comment on the result.

4. Find the total yardage and total value of cotton piece-goods exported in 1925 from *Stat. Abs.*, Table 220, pp. 347–9. Calculate the average price per square yard.

Now compute the average prices of each of the seven kinds. "Weight" these averages (1) with the number of hundred-million square yards of each kind, (2) with the values to the nearest £100,000,000, and find the "weighted average" price. Explain why the result of (1) agrees very closely with average already calculated, while the result of (2) is 8% too large.

On Chapter V

1. Make diagrams of the type on p. 37 of the wages shown in Part II, Ch. VI, p. 168.

2. Make circular diagrams (as on p. 47) of the main items of revenue and expenditure in the first and last years shown in Part II, Ch. X, p. 220.

3. From the Tables of Imports and Exports (*Stat. Abs.*, Tables 209 and 210) make the following diagrams, for the years 1912 to 1926 :—

(i) Of Imports from Foreign Countries, British Countries and Total.
(ii) Of Exports to Foreign Countries, British Countries and Total.
(iii) Total Imports and Exports.
(iv) Imports and Exports to and from British Countries.

4. From Table 220 (*Stat. Abs.*, pp. 346–7) make three diagrams of the value and quantity of cotton grey unbleached piece goods exported, (1) representing 100 yds. and £1 by same unit, (2) making the lines start together, (3) making the lines end together.

5. Represent the numbers in the table on p. 208, Part II, Ch. IX, by a diagram.

6. Treat one or more of the columns of the prices in the table on p. 140, Part II, Ch. IV, by the method of p. 42.

On Chapter VI

[Use round numbers throughout and pay attention to clearness of meaning and legibility.]

1. Make a table of Exports (*Stat. Abs.*, Table 210), grouping together the Foreign Countries in Europe, Asia, Africa and America, and showing British Countries in 5 groups, from 1912 to 1926.

2. Make a table combining imports and exports of bullion and specie with those of merchandise (*Stat. Abs.*, Tables 208, 210, 222) for the aggregates of Foreign and British Countries separately in 1913, 1920 and 1926.

3. Make two or more tables showing the trade and shipping of Bristol in 1913 and 1925 (*Stat. Abs.*, Tables 194, 195, 197, 198 and 212).

4. Re-arrange the Table on p. 190, Part II, making 1913, 1921 and 1925 the heads of columns (interchanging columns and rows), merging together some of the benefits and stating the total expenditure, for Great Britain only.

5. Combine the statistics relating to pig-iron and ferro-alloys (*Stat. Abs.*, Tables 171, 214 and 220) so as to show the amount available for use in the United Kingdom in 1913 and in 1925.

On Chapter VII

1. Find by sampling the number of words (1) in a full line of this book, (2) in lines including those at the beginning and end of a paragraph. Hence estimate the number of words in a page containing 37 lines. Calculate the precision of your estimate, and verify it by counting the number of words in a number of pages.

2. Make a similar estimate for the average number of letters in a word. Also by taking, say, 1000 words, find the frequency of words of different lengths. Estimate the precision of your results, and verify it by tabulating a large number of consecutive words.

3. Find the ratio of the number of commas to the number of full stops in this or any other book.

4. Find whether the digits 0 to 9 are uniformly distributed through the table at the beginning of Chap. IV, Part I.

On Chapter VIII

1. Apply the rules of criticism given to—
   (1) the various statistics relating to Persons in Receipt of Relief (England and Wales) (*Stat. Abs.*, Tables 54, 55, and 59).
   (2) The Post Office statistics, pp. 250–5.
   (3) the categories of expenditure by Local Authorities in England and Wales, Table 116, p. 173 with reference also to Table 117.
   (4) The Income Tax categories, Table 102, p. 147.

2. How far can the statistics of (1) wages, (2) consumption of meat, (3) value of exported manufactures, be regarded as tests of national progress?

3. Criticize *Stat. Abs.*, Table 216, "Home Consumption per head," from the point of view of paragraph 5.

4. Criticize the line of percentage increase at the bottom of *Stat. Abs.*, Table 80, p. 95, in the light of paragraph 4.

On Chapter IX

1. Verify the averages in the table in paragraph 3. Deduce the number of miles of track and of route. Show that ton-miles per engine-hour, divided by wagon-miles per engine-hour would equal the average full-and-empty wagon load; and that wagon-miles per engine-hour, divided by train-load would give train-miles per engine- (train and shunting) hour.

2. Consider what data would give the best information for any business or institution with which you are acquainted.

3. Make a blank card suitable for entering details as to a workman applying at a Labour Exchange.

4. Draw up a blank schedule suitable for tabulating details of working-class expenditure.

5. Required to describe the housing accommodation of a district. How would you proceed and what blank forms would you use (1) if you had legal power of entry and measurement, (2) if the inquiry was on a voluntary basis?

PART II

On Chapters I and II

1. Calculate some of the *birth-rates* in Table 7 (*Stat. Abs.*, p. 7), from the number of births and from the population stated in Table 5.

2. Estimate the population of Scotland for each year from 1911 to 1921, using only the data of Table 4, and compare your results with those of Table 5.

3. Work out from the Census Report for your county the density of population in as much detail as possible in your neighbourhood.

4. From the statistics of population, births and deaths (Tables 4 and 7), find the excess of emigrants over immigrants for Scotland between 1911 and 1921, and compare your result with Table 6. [In 1911, births 121,850 and deaths 71,732.]

5. With the help of a diagram estimate the actual and relative number of men between the ages 32 and 38 in table on p. 102 above; and also the number of children between 7 and 14. (The whole population is given on p. 88.)

6. Find the actual numbers in various occupations from the per mille table on p. 94, and state in what cases the absolute numbers have increased while the relative numbers have diminished.

7. How is it that the infant mortality rate is lower than the death-rate between 0 and 1 years?

8. In Table 7 (*Stat. Abs.*, p. 7) calculate the population of the United Kingdom in 1921 and 1926 from the number of births and the birth-rate, as accurately as these data allow, and compare with Tables 4 and 5.

9. Find the corrected death-rate for District B to compare with District A as standard, by both the methods described on pages 112–14, from the following data. Find also the general uncorrected death-rates.

		Years 0–5.	Years 5–15	Years 15–55	Years 55–
District A.	Relative number of persons .	114	110	670	106
	Death-rates	4	3	7	60
B.	Relative number of persons .	136	125	619	120
	Death-rates	38	3	6	55

On Chapter III

1. Make the tables corresponding to those in paragraphs 3 and 4 for the years 1920 to 1926. (*Stat. Abs.*, Tables 207, 222, 223).

2. Make a table of the excess of imports over exports (including bullion) for the years 1912, 1913 and 1920–6, and express this excess year by year as a percentage of the total of imports and exports. On the same diagram show the numbers in this table, and the total tonnage of vessels registered as belonging to the United Kingdom (Tables 207, 222, and 202).

3. Draw diagrams showing (1) total value of imports and total tonnage of ships entered with cargoes, and (2) total value of exports and total tonnage of ships cleared with cargoes for the years 1920 to 1926. (*Stat. Abs.*, Tables 207, 192).

4. Draw smoothed diagrams (as Diagram III, p. 42, above) representing the table of external trade on p. 127 above.

5. Illustrate the process described in paragraph 8 by computing the average prices of imported meat (*Stat. Abs.*, Table 214, pp. 312–13) in 1913 and re-valuing the quantities imported in 1920 and 1926 at these prices. Obtain the totals of the values as declared and of the re-valuation so far as the data allow. Work to three significant figures only.

6. What proportions of the totals of Classes I, II, III and of the total are contained in the detailed list in *Stat. Abs.* (Table 221, pp. 355–9) in 1926? The totals are given in the last line of Table 213.

7. Draw a diagram illustrating the increase of steam and motor ships relative to sailing ships from *Stat. Abs.*, Table 192, pp. 268–9.

On Chapter IV

1. Make index-numbers of the prices of imported meat from the figures obtained in Exercise 5 on last chapter.

2. Calculate index-numbers for 1880–4, 1890, 1900, 1913 and 1927, for the eight commodities together (wheat to coal) shown in paragraph 4, (1) taking 1865–9 as the basis, (2) taking 1875–9 as the basis, (3) taking 1900 as the basis. In each case re-write the index-numbers so that the number for 1913 is 100. Comment on the differences shown.

[Note.—So few commodities are, of course, insufficient for establishing a general index-number.]

3. Transfer Sauerbeck's index-numbers from gold values (in which they are given on p. 140) to silver values.

4. From the *Stat. Abs.* (Tables 170, 214, 220) make a table and diagram comparing the prices of pig-iron imported, exported and produced.

On Chapter V

1. Make a table for 1912–14 and 1919–26 from the *Stat. Abs.* (Tables 214, 220, 221) showing the value of imported raw cotton (less re-exports) as compared with the value of exported cotton goods. Assuming that 80% of imported cotton is used for the foreign trade, find the value added by manufacture year by year.

2. The Census of Production shows that the value of the output of cotton factories in 1924 was £82,380,000 more than that of cost of materials used. The value of cotton imported and retained that year was £107,960,000, and of exported cotton manufactures was £199,162,000. If these statements are consistent with the 80% assumption of the last exercise, deduce the value of materials used (coal, etc.) other than raw cotton.

3. From the Table on p. 152 compute the net output numbers employed, net output per head, horse-power, and horse-power per wage-earner in the United States in 1907 and 1924, assuming uniform movement from 1904 to 1909 and

from 1923 to 1925. Then compare the changes in these categories 1907 to 1924 with those in the United Kingdom (using the dollar columns).

On Chapter VI

1. The wages of 2,000 men were increased 1*d.* per hour and the normal week was decreased 3 hours. If before the change the rate was 20*d.* and the week 50 hours, compute the effect that would be shown in a "change of wages" table.

2. If average weekly wages in Textiles, Agriculture, Building, and Engineering had been respectively 15*s.*, 13*s.*, 25*s.* and 27*s.*; and the relative numbers employed 5, 10, 2 and 3 in 1880, compute the change per cent. for the 4 groups together in 1890, 1900 and 1908 from the index-numbers in paragraph 9, (1) assuming no change in the relative numbers, (2) assuming that the numbers changed gradually till in 1908 they were 5, 7, 3, 5.

3. If average wages rise 20%, and the retail purchasing power of money rises 10%, how much do average real wages rise?

4.

Wages. Grade.	Number of men. Year 1.	Year 2.
46*s.*–48*s.*	25	15
48*s.*–50*s.*	25	25
50*s.*–52*s.*	25	35
52*s.*–54*s.*	25	25

Find the maximum and minimum change possible in average wages consistent with promotions as shown in this table, assuming that no man's wage was reduced.

5. Compute the lines for lads and girls on page 168 on the assumption that all receiving less than 5*s.* were half-timers (none earning less than 2*s.* 6*d.*) and supposing each pair of half-timers replaced by one full-timer at their joint wages.

On Chapter VII

1. Make diagrams illustrating the table on p. 176.

2. For lines A and B of the same table take decennial averages for 50 periods beginning 1851, 1852, to 1900, and represent the result in a diagram. Comment on the result.

3. Compute column D counting B_2 as twice as important as B_1.

4. Which of the lines in the table on p. 178 can be calculated from the other lines?

5. Make a diagram showing the general percentage unemployment, as shown on p. 179, for every month from January 1897 to December 1909.

If seasonal changes are eliminated, which was the worst month in 1904–5?

6. From the Table on p. 183 compute the percentage unemployed in "Other Industries" each quarter, taking the total number insured in this group as 3,703,000 in July 1927, and assuming that this number had increased 1% per annum.

On Chapter VIII

1. Express the expenditures shown in the table on p. 189 as percentages of the total expenditure.

2. What information does the *Stat. Abs.* contain as to working-class savings?

3. Check the percentages shown on pp. 194–5.

4. How far can the higher expenditure shown (pp. 191–5) for English incomes than for French or German over 40*s.* be accounted for by the larger number of children?

5. Re-compute the index-numbers for the United States in June 1925 (p. 197), giving each category the same importance as in the United Kingdom (omitting furniture).

On Chapter IX

1. From the table on p. 201 compute the average income per head at each date, and by using the *Statist* or Sauerbeck's

index-numbers, p. 140, eliminate roughly the influence of the change of purchasing power of money. (From p. 140 the index-numbers for 1860, 1870, and 1880 can be taken as 142, 145 and 127.)

2. From the table on p. 210 write down the number of persons with incomes *at or above* certain incomes in both years. Next write down the logarithms both of incomes and numbers in each year. Plot the logarithms as rectangular co-ordinates. The results for each year should be approximately straight lines.

Now divide the income scale in 1924–5 by 1·70 to allow for the change in purchasing power of money and re-draw the line on this basis.

3. Compare the changes shown on p. 213 in national income, aggregate and per head, in the United States and the United Kingdom with the figures of net output (p. 152).

On Chapter X

1. If the whole of indirect taxation in 1913–14 (p. 226) were borne by working-class families and others with incomes below £160, and the whole of direct taxation by income-tax payers, and if the two classes consisted respectively of 7,000,000, and 1,000,000 families and their aggregate incomes of £1,000 mln. and £800 mln., calculate the burden per family in each case and the proportion of taxes to income in each case. [Omit Post Office and Crown Lands, etc.]

2. Estimate the aggregate income of Great Britain in 1907–8 from p. 222 on the hypothesis that among persons where the rent is

less than £25,	the average family income is		8	times the rent
£25 to £50	„	„	10	„
£50 to £80	„	„	12	„
£80 to £500	„	„	15	„
£500 and over	„	„	20	„

Miscellaneous Examination Questions

1. In the following table the density is measured by the number of persons to the square mile and the population in each line is given correct to the nearest thousand :

Density of districts.	Inhabitants. 000's.
50–100	15
100–200	50
200–300	55
300–400	40
400–500	25
500–600	10
600–700	5

Estimate the area of the aggregate of the districts and also the density of the aggregate. What is the maximum density that is consistent with the data ?

2. 5,300 children under 15 years old form 31% of a population. If the number of children is given to the nearest 100 and the percentage to the nearest unit, to what degree of accuracy can the population be estimated from this statement ?

3. Of 743,000 cwt. the average price of 21% was 58*s*. and of the rest 74*s*. per cwt. If the price is stated to the nearest shilling and the quantity to the nearest 1,000 cwt., find the greatest and least amounts that the whole can have cost.

4. 465,000 persons were employed in coal-mines in November 1918, whose average earnings were £13 in four weeks. Of these, 139,000 were coal-getters, with average earnings £15 10*s*. Calculate the average earnings of other operatives, supposing the figures exact, and find also the minimum wage consistent with the data, if the numbers are given only as the nearest thousand and the earnings as the nearest 10*s*.

5. The average wages of two groups, containing n_1 and n_2 persons respectively, are a_1 and a_2, and the average of the two groups merged is A. $n = n_1 + n_2$. A, n, n_1 and a_1 are known approximately, but each may be 1% in error in excess or defect. A is less than a_1.

Show that the greatest value of a_2, consistent with this statement, is obtained when A and n are taken as great, and a_1 and n_1 as small as possible, and work out the result when A = 40, a_1 = 45, n = 73,700, n_1 = 30,600.

6.

Occupied in Hat Manufacture

	All.	On Piece-rates.	On Time-rates.
	%	%	%
Males . . .	63	53	79
Females . . .	37	47	21
	100	100	100

From this table find the percentages paid piece-rates (1) of all employed, (2) of all males, and (3) of all females.

7. If, in an industry employing men and women, men form 40% of all employed, and men paid time and piece-rates form respectively 20% and 45% of all paid time and piece-rates, find the proportion of all employed, and also of men and of women employed, who are paid piece rates.

8.

Cotton Trade

Number of workpeople.		Total wages.	
August 1922.	Increase over August 1921.	August 1922.	Decrease from August 1921.
89,026	6·5%	£168,505	6·6%

Compute the numbers and wages in August 1921 and the change in the average wage.

9. Sauerbeck's index-numbers for 45 commodities in 1916 (the averages of 1867–77 being taken as 100) were 107, 121, 114, 132, 128, 163, 131, 168, 138, 154, 148, 157, 169, 148, 153, 100, 93, 84, 68, 135, 166, 154, 173, 159, 125, 197, 100, 104, 172, 161, 163, 159, 101, 71, 174, 160, 104, 114, 119, 135, 96, 86, 128, 183, 202 respectively.

Find the arithmetic mean, median, quartiles and quartile deviation of these numbers.

10. Find the average, median, mode and one measurement of deviation in the following frequency table:

AVERAGE SIZE OF FAMILY IN 128 DISTRICTS

Persons per family.	Number of districts.	Persons per family.	Number of districts.
3·5 to 3·6	1	4·3 to 4·4	32
3·6 ,, 3·7	1	4·4 ,, 4·5	23
3·7 ,, 3·8	2	4·5 ,, 4·6	10
3·8 ,, 3·9	1	4·6 ,, 4·7	11
3·9 ,, 4·0	2	4·7 ,, 4·8	2
4·0 ,, 4·1	3	4·8 ,, 4·9	0
4·1 ,, 4·2	8	4·9 ,, 5·0	1
4·2 ,, 4·3	30	5·0 ,, 5·1	1

11. A weighted average, Q, is obtained by applying approximate weights w_1, w_2 . . . to known quantities q_1, q_2 . . . Show that, if the weights are slightly modified so as to be $w_1 + e_1$, $w_2 + e_2$ . . ., Q becomes $Q + S(q_1 - Q)e_1/Sw$ approximately.

12.

IMPORTS INTO THE UNITED KINGDOM

		Quantities.			Values.		
		1913.	1918.	1919.	1913	1918.	1919.
	Unit.	000,000's.			£000,000's.		
Wheat	cwt.	106	58	71	44	53	68
Beef	cwt.	9	8	6	16	36	30
Cotton	lb.	2,174	1,489	1,958	79	150	190
Wool	lb.	801	413	1,043	34	36	97

Show the change in volume of the total importation of these four commodities when weights are based on (*a*) 1919 values, (*b*) 1913 values.

13. Point out the ambiguities or errors in the following statements, and if possible word them correctly:

(1) The following table shows the increase of the price of vegetables in Germany (*Metalarbeiter-Zeitung*):

	1914.	Nov. 1917.	Per cent. increase.
	Pfennigs.		
Potatoes . . .	2·5	8	220·00
Carrots . . .	3	12	300·00
Kohlrabi . . .	1·5	8	433·33
White cabbage . .	3	25	733·33
Onions . . .	6	25	316·66
Average increase . .			400·66

(2) " The increase in prices in Stockholm was 110% from July 1914 to December 1917; in the first year it was 26%, in the second a further 12, in the third year up to July 35 and during the last half-year 35."

(3) The average size of a family decreased from 4·5 to 3·5 in 10 years, and this explains the slower growth of the population.

(4) While rates of wages increased 10%, income subject to tax increased 15%; hence wage-earners are losing relatively.

(5) In the two periods 1902–6 and 1906–12 the purchasing power of the sovereign fell equally, for the index-numbers of prices were 69, 77, and 85 in 1902, 1906, and 1912 respectively.

(6) New-laid eggs were sold at 6 to the shilling, imported at 10 to the shilling. The average was therefore 8 to the shilling.

(7) In 1901 and 1911 the populations of New Zealand were 20·0 and 22·6% of those of Australia, a relative increase in 10 years of 11·30%.

(8) In one period, money-wages rose 10% and prices fell 5%, and in the next wages fell 8% and prices rose 7%, so that in all real wages neither rose nor fell.

(9) It is observed that married men live longer than unmarried; hence we conclude that marriage is conducive to health.

(10) The wages of each group rose 5% and therefore the wages of the aggregate rose 5%.

APPENDIX II

Selected List of Books of Reference

Jevons—Investigations in Currency and Finance. Macmillan.
Giffen—Economic Inquiries and Studies. G. Bell & Sons.
Goschen—Essays and Addresses on Economic Questions. Arnold.
Bowley—Official Statistics. Oxford University Press.
Carr-Saunders and Jones—A Survey of the Social Structure of England and Wales. Oxford University Press.

Government Publications.

Government publications are either issued by one of the Ministries or Departments without a code number, or as "Command Papers," or as "House of Commons Reports or Papers." In the first case it is necessary to quote the abridged title of the report and the date, when ordering; in the second case the code number is sufficient, *e.g.* Cmd. 1774 (prior to 1900 C. was the prefix, from 1900 to 1918 Cd., and subsequently Cmd.); in the third case such an abbreviation as H.C. 365 of 1906 is sufficient.

Periodical Publications.

The reference number is for the last issued prior to June or July 1928.

	Number.	Price. s.	d.
Guide to Current Official Statistics, Vol. VI, 1927 [1] .		1	0
Statistical Abstract for the United Kingdom, 1926 .	Cmd. 3084	6	6
„ „ „ British Overseas Dominions, 1923	Cmd. 2738	7	6
Abstract of Labour Statistics, 1926 . . .	Cmd. 2740	4	0
Report of the Ministry of Labour for the year 1927 .	Cmd. 3090	2	6
Annual Statement of the Trade of the United Kingdom for 1926, Vol. I		14	0
Annual Statement of the Trade of the United Kingdom for 1926, Vol II		25	0
Annual Statement of the Trade of the United Kingdom for 1926, Vol. III		25	0
Annual Statement of the Trade of the United Kingdom for 1926, Vol. IV		25	0
Monthly Trade and Navigation Accounts. The twelve months ending December 1927 . .		4	6
Quarterly Trade and Navigation Accounts for Foreign Countries, etc.		5*s.* to 7	6
Navigation and Shipping in 1926 . . .		15	0

[1] See Volume II, 1923, for publications between 1900 and 1922.

	Number.	Price. s.	d.
League of Nations Monthly Bulletin		1	6
Ministry of Labour Gazette. Monthly			6
Board of Trade Journal			6
The Registrar-General's Statistical Review of England and Wales. Text for 1925		5	0
The Registrar-General's Statistical Review of England and Wales. Part I. Medical for 1926		15	0
The Registrar-General's Statistical Review of England and Wales. Part II. Civil for 1926		5	0
Annual Report of the Secretary for Mines for 1926		5	6
Agricultural Statistics, 1927, Part I		1	3
„ „ 1926, Part II		1	3
Railway Statistics. Monthly	2*s.* 6*d.* to	3	6
Finance Accounts of the United Kingdom for 1926–7	H. C. 71	2	0
Report of the Commissioners of Inland Revenue for 1926–7	Cmd. 2989	2	0
Report of the Commissioners of Customs and Excise for 1926–7	Cmd. 2960	3	0
Special Reports.			
Census of Production, 1907	Cd. 6320	7	6
Census of Production 1924. (*To be issued.*)			
Census of Population: England and Wales, 1911:			
General Report	Cd. 8491	4	6
Administrative Areas, Vol. I	Cd. 6258	5	4
Ages and Conditions as to Marriage, Vol. VII	Cd. 6610	4	11
Occupations and Industries, Vol. X. Two parts and Appendix	Cd. 7018	8	0
	7019	6	3
	7660	3	0
Census of Population: England and Wales, 1921:			
General Report		5	0
General Tables		13	0
Occupations		24	0
Industry Tables		47	6
Census of Population: Scotland, 1921: Preliminary Report	Cmd. 1473	1	8
Ages, Conjugal Condition, Birthplaces, Housing, etc., Vol. II		20	0
Occupations and Industries, Vol. III		30	0
Census of Northern Ireland, 1926: Preliminary Report		1	0
Census of Population of Irish Free State: Preliminary Report			3
Public Health and Social Conditions, 1909	Cd. 4671	5	0
Cost of Living in the United Kingdom, 1912	Cd. 6955	4	11
Decennial Supplement to Registrar-General's Report, 1921: Part I. Life Tables		2	0
Decennial Supplement to Registrar-General's Report, 1921: Part II. Occupation Mortality, Fertility and Infant Mortality		7	6

	Number.	Price. s.	d.
Committee on Industry and Trade: Overseas Markets		6	0
Committee on Industry and Trade: Survey of Industrial Relations		5	0
Committee on Industry and Trade: Factors in Industrial Efficiency, Part I		5	0
Committee on Industry and Trade: Factors in Industrial Efficiency, Part II		3	6
Committee on Industry and Trade: Survey of Metal Industries		5	0
Committee on Industry and Trade: Survey of Textile Industries		3	6
Royal Commission on the Coal Industry, 1925: Vol. I Report	Cmd. 2600	1	0
Committee on National Debt and Taxation: Report	Cmd. 2800	7	6
Unemployment Insurance Committee: Report		1	0

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